A

Guide to the Lottery;

OR, THE

LAWS OF CHANCE.

[Price Two Shillings.]

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A
Guide to the Lottery;
OR, THE
LAWS OF CHANCE

Laid down in a plain and intelligible Manner, wherein is ſhewn the Probabilities ariſing from any propoſed Circumſtance of Play; applied to the Solution of a Variety of curious Queſtions relating to Cards, Dice, Lotteries, &c.

LIKEWISE

The whole Buſineſs of inſuring Tickets in the State Lottery clearly explained, the ſeveral Advantages taken by the Office-keepers pointed out, and an eaſy Method given, whereby any Perſon may compute the Probability of his Succeſs upon purchaſing or inſuring any particular Number of Tickets, with a Table of the Price of Inſurance for every Day's drawing in the enſuing Lottery, another Table, containing the Number of Tickets a Perſon ought to purchaſe to make it an equal Chance to have any particular Prize; with ſeveral other curious Tables.

TO WHICH IS ADDED

A COMPANION FOR THE DRAUGHT-PLAYER,

CONTAINING

Thirty ſelect Games of DRAUGHTS, ſhewing the Manner of moving the Pieces to the beſt Advantage; together with ſeveral critical Situations to win Games, and fine Strokes, never before publiſhed: being the Reſult of the Practice and Obſervations of ſome of the firſt Players.

BY W. PAINTER.

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INTRODUCTION.

IN all games the number of chances for winning and losing must be considered, from whence the probabilities of success will be obtained. Now the probability of an event happening is to that of its failing, as the number of different ways by which it can happen to the number of different ways by which it can fail: thus, if I have 3 chances to win 4 pounds, and 3 chances whereby I may not win any thing, my expectation in this case will be worth 2 pounds, it being an equal chance whether I get 4 pounds or nothing; and consequently, if a person was to purchase my expectation, he ought to give me 2 pounds for it. Again, suppose a person holds a certain sum of money in each hand, and I am to choose which hand I will, I say the value of my expectation is in this case half the sum of money in both hands, for suppose 7 pounds in one hand, and 9 in the other, then it is evident I have an equal chance for either 7 pounds or 9, therefore my expectation is evidently worth 8 pounds.

To find the probability of an event happening, proceed thus: add the number of chances for the event happening to those of its failing, and that

ſum compared with the chances for the event happening or failing will expreſs their reſpective probabilities for gain and loſs.—Suppoſe an event has 3 chances to happen, and 2 to fail, the ſum of 3 and 2 is five, then the probability of the event happening will be as 5 to 3, and that of its failing as 5 to 2.

The probability of two or more events happening is equal to the product of the probabilities whereby thoſe events may happen ſingly.

Queſtion 1.

Suppoſe with a common die of ſix faces I undertake to throw the ace twice ſucceſſively, what is the probability of my ſucceſs.

Solution. The probability of throwing an ace the firſt time is as 6 to 1, and that of throwing it the ſecond time as 6 to 1, and the product of 6 by 6, *viz.* 36 are the number of chances againſt me for throwing an ace twice ſucceſſively, that is as 36 to 1.

Queſtion 2.

Suppoſe there are 3 parcels of 4 cards each, containing an ace, king, queen, and knave, what are the odds that in taking one from each I ſhall take the three aces?

Solution. The odds for drawing an ace from any one heap are as 4 to 1, that of drawing two aces

aces as 4 multiplied by 4, or 16 to 1, and that of drawing a third ace, as 16 multiplied by 4, or 64 to 1.

Queſtion 3.

Suppoſe there is a heap of 13 cards of one ſuit, what is the probability that in drawing three of them they ſhall be the king, queen, and knave?

The ſolution of this queſtion differs ſomething from the two former, here being but one heap, whereas in the other there were three, and each heap contained the ſame number of cards at each drawing; but in this the number of things are leſſened at each drawing, and are 13, 12, and 11; theſe multiplied together, produce 1716, therefore the odds are as 1716 to 1.

N. B. The ſolution will be the ſame whether the cards are drawn one at a time or all together.

Queſtion 4.

Let there be a heap of 10 cards, of which 4 are diamonds and 6 clubs, what is the probability that, in drawing two of them, they ſhall be both diamonds?

Solution. Suppoſe them drawn one at a time: now the number of chances for this event to happen once are 4, and thoſe for its failing 6, therefore the probability that this event ſhall happen the

 firſt

first time is as 10 to 4, that is, the probability of drawing a diamond the first time; if a diamond be so drawn we have 9 cards left, 3 of which are diamonds, and the probability of drawing a diamond the second time will be as 9 to 3; now the number of chances for the event to happen are 4 and 3, these multiplied together produce 12; the number of chances against it are 10 and 9, whose product is 90, whence the probability of drawing 2 diamonds successively is as 90 to 12, or as 15 to 2.

Question 5.

Let it be required to find the probability of drawing 2 clubs from the same heap.

Solution. There being 10 cards as before, 6 of which are clubs, the probability of drawing a club the first time will be as 10 to 6; if a club be so drawn we shall have 9 cards left, 5 of which will be clubs, and the probability of drawing a club the next time, will be as 9 to 5; now the number of chances for succeeding are 6 and 5, whose product is 30, and those against it are 10 and 9, whose product is 90, therefore the odds for drawing 2 clubs successively are as 90 to 30, or just as 3 to 1.

Question 6.

Again, let it be required to find the probability of drawing all the diamonds first.

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In the ſolution of queſtions of this nature where there is but one parcel or ſet of things concerned, it is evident that the number of them continually decreaſe by one at each drawing, and are taken to as many terms as there are number of drawings, and then the reſpective chances for the happening and failing of the ſeveral events being multiplied together, produce the probability of all the events happening; we ſhall therefore for the future place the number of chances for the happening of an event above a line, and thoſe againſt it below, when it muſt be obſerved to multiply all the numbers found above the line into each other, and likewiſe all thoſe below into each other, and the products will ſhew the probability of all the events happening: Thus in the preſent queſtion the number of events or drawings are 4, and it is required to draw all the diamonds out of the heap of 10 cards at 4 drawings; now the probability of drawing a diamond the firſt time was found to be as 10 to 4, which I expreſs thus $\frac{4}{10}$; the probability of drawing a diamond the ſecond time was found to be as 9 to 3, which I expreſs thus $\frac{3}{9}$; if two diamonds be ſo drawn, we ſhall have 8 cards left, 2 of which are diamonds, and the probability of drawing a diamond the next time will be as 8 to 2, or $\frac{2}{8}$; if this be effected, we ſhall have 7 cards left, 1 of which is a diamond, therefore the probability of taking a diamond the fourth time will be as 7

to

to 1, expressed thus, $\frac{1}{7}$; now collecting all these probabilities together, we have $\frac{4}{10}\ \frac{3}{9}\ \frac{2}{8}\ \frac{1}{7}$, then multiplying the lower numbers together, produce 5040, and multiplying the upper ones together, make 24, so the required probability is as 5040 to 24, or exactly as 210 to 1.

Question 7.

Let there be 10 cards taken as before, *viz.* 6 clubs and 4 diamonds, but let them be divided into two heaps, each containing 3 clubs and 2 diamonds; required the probability of drawing a diamond from each heap.

Solution. The probability of taking a diamond from one heap will be as 5 to 2; now as the drawing or not drawing of a diamond from one heap does not affect the drawing one from the other, therefore the probability of taking a diamond from the other heap will be also as 5 to 2, and the probability of both these events happening will be thus expressed $\frac{2}{5}\ \frac{2}{5}$, now the product of 5 by 5 is 25, and that of 2 by 2 is 4, so the probability is as 25 to 4.

Question 8.

Suppose there is a lottery in which are 100 tickets, containing 24 capital prizes, what is the probability that, in taking three tickets, I shall have 3 of those prizes?

Solution,

Solution. As the queſtion requires three of thoſe particular prizes, it is no matter what other prizes are in the lottery beſides theſe principal ones; therefore, all the reſt of the tickets may be eſteemed as blanks, and the probability of having one of thoſe prizes will be as 100 to 24, which is the ſame as 25 to 6; if one of thoſe prizes be ſo drawn, the probability of having another of them will be as 99 to 23, and that of having a third as 98 to 22; now collecting all theſe probabilities together, we have $\frac{6}{25}\,\frac{23}{99}\,\frac{22}{98}$, and multiplying the lower numbers together, produces 242550, and the upper ones 3036, ſo the required probability is as 242550 to 3036, or about 80 to 1.

Queſtion 9.

Suppoſe there is a heap of 12 cards, containing 8 clubs and 4 diamonds, required the probability that in drawing 2 of them, one of the two ſhall be a diamond.

The ſolution of this queſtion differs from thoſe of the foregoing ones, becauſe here we only require one of the ſeveral things drawn to anſwer the conditions of the queſtion, and the readieſt way to diſcover this, will be to find the probability of the contrary happening, that is, to find the probability of drawing two clubs ſucceſſively, and ſubtracting that from the whole number of chances for the happening and failing of the event, the

remainder

remainder will be the number of chances for drawing one diamond at least. Thus the probability of drawing 2 clubs successively will be $\frac{8}{12}, \frac{7}{11}$, the product of 12 by 11 being 132, and that of 8 by 7, 56; so we have 56 chances for drawing 2 clubs successively, and 132 against it; therefore, subtracting 56 from 132, there remains 76, the number of chances for drawing one diamond, and the required probability as 132 to 76, or as 33 to 19.

Question 10.

Let there be a lottery of 500 tickets, in which there are 4 particular prizes, what is the probability that in taking 3 tickets I shall have one of these prizes?

Solution. First find the probability of the three tickets being all blanks thus: consider all the tickets, except the 4 particular ones, to be blanks, whose number will be 496, then the probability of these three tickets being all blanks, will be $\frac{496}{500}, \frac{495}{499}, \frac{494}{498}$; the product of the upper numbers is 121286880, and that of the lower ones 124251000, their difference 2,964,120, is the number of chances for drawing one prize, and the probability will be as 124,251,000 to 2,964,120, or nearly as 42 to 1.

Question 11.

Let there be an example in the ensuing lottery, in which are 50,000; tickets amongst these

are

are 250 prizes of 50l. each, what are the odds, that, in taking 2 tickets, I ſhall have one prize of 50 l.?

It being of no conſequence to the ſolution of this queſtion what other prizes there are in the lottery, therefore all the reſt of the tickets, except theſe 250 may be eſteemed as blanks. Now 250 taken from 50,000, leaves 49,750, which is to be conſidered as the number of blanks in this queſtion; therefore $\frac{49750}{50000}$, $\frac{49749}{49999}$ expreſſes the probability of the two tickets being blanks or prizes under 50l. the upper product is 2,475,012,750, the under one 2,499,950,000, their difference 24,937,250 are the number of chances for having one prize, and the required probability, as 2,499,950,000 to 24,937,250, or nearly as 100 to 1.

A Table, shewing the Probability of having one Prize at least of the several Prizes in the ensuing Lottery, which begins drawing February 12, 1787, by purchasing a certain Number of Tickets.

Numb.	£ 20,000.	10,000.	5,000.	2,000.	1,000.	500.	100.	50.	Any Prize above 20l.
3	8333 to 1	5555	3333	1667	757	555	166	66	39
4	6244 1	4166	2498	1249	568	416	125	50	29
5	4983 1	3333	1998	999	455	333	100	40	23
6	4166 1	2778	1666	833	379	277	83	33	19
7	3571 1	2381	1428	714	325	238	71	28	17
8	3122 1	2083	1249	625	284	208	62	25	$14\frac{8}{10}$
9	2778 1	1852	1111	556	252	185	55	22	13
10	2491 1	1666	999	560	227	166	50	20	$11\frac{8}{10}$
11	2273 1	1515	909	455	206	151	45	18	$10\frac{8}{10}$
12	2083 1	1389	833	416	189	139	41	16	10
13	1923 1	1282	769	385	175	128	38	15	9
14	1786 1	1190	714	357	162	119	35	14	$8\frac{4}{10}$
15	1666 1	1110	666	333	151	111	33	$13\frac{1}{3}$	$7\frac{9}{10}$
16	1562 1	1041	625	317	142	104	31	$12\frac{1}{2}$	$7\frac{9}{10}$
17	1470 1	980	588	294	133	98	29	$11\frac{7}{9}$	7
18	1389 1	926	555	278	126	92	27	11	$6\frac{6}{10}$
19	1316 1	877	526	263	119	87	26	$10\frac{1}{2}$	$6\frac{2}{10}$
20	1245 1	833	499	250	113	83	25	10	$5\frac{12}{100}$

The

The Uſe of the preceding Table.

The firſt column contains the number of tickets, the reſt of the columns contain odds againſt having any reſpective prize, ſpecified at the top of the table: for inſtance, ſuppoſe I purchaſe 6 tickets in the enſuing ſtate lottery, I deſire to know the probability of my having one of theſe tickets a 1000l. prize? I look in the firſt column, under Numb. for 6, and againſt it, under 1000, I find 379, by which I ſee that it is 379 to 1 that I have a 1000l. prize out of the 6 tickets.

Again, Suppoſe I purchaſe 12 tickets, what are the odds againſt me that I have one prize of 50l.? I look for 12 in the firſt column, and againſt it in a right line under 50, I find 16, which ſhews that it is as 16 to 1 that I have a prize of 50l. out of the 12 tickets.

To find the Value of a Perſon's Expectation on any particular Chance.

Queſtion 12.

Suppoſe that out of a heap of 10 counters, of which 7 are black and 3 red, I am required to draw 2, and in caſe one of the two be a red one, I am to receive the ſum of 5 ſhillings, what is the value of my expectation?

Solution. Find as before the probability of drawing two black counters ſucceſſively thus: the

probability of drawing a black counter the firſt time will be as 10 to 7, that of taking a black one the ſecond time as 9 to 6, or as 3 to 2, becauſe both the 9 and the 6 may be reduced to lower quantities, they being both diviſible by 3; then collecting theſe two probabilities together, we have $\frac{7}{10}$ $\frac{2}{3}$; the product of 10 by 3 is 30, and that of 7 by 2 is 14; therefore the probability of drawing two black counters ſucceſſively is as 30 to 14, that is, as 15 to 7, becauſe 30 and 14 may both be divided by 2; now 7, the number of chances for drawing two black counters ſucceſſively, being taken from 15, the whole number of chances, leaves 8 for the number of chances for taking one white counter, ſo the required probability is as 15 to 8; now to find the value of the expectation, there being 15 chances in the whole, 8 of which are in my favour, therefore the value of my expectation will be found by dividing 5 ſhillings into 15 parts, and taking 8 of thoſe parts, I ſhall have the value required; thus in 5 ſhillings are 60 pence, which divided by 15 quotes 4, and this multiplied by 8 gives 32 pence, or two ſhillings and eight pence. Or the general method is to multiply the ſum by the number of chances in your favour, and that product, divided by the whole number of chances, gives the value; thus 5 ſhillings multiplied by 8 gives 40, and this divided by 15 quotes 2 ſhillings and 8 pence, the ſame as before.

Queſtion

Queſtion 13.

There are two parcels of three cards each, the firſt containing king, queen, and knave of hearts, the ſecond parcel the king, queen, and knave of diamonds: now ſuppoſe I am promiſed the ſum of 3 guineas, in caſe that in taking a card out of each parcel I ſhall take either the king of hearts or the king of diamonds, required the value of my expectation?

Solution. The probability of not taking a king from the firſt parcel is as 3 to 2, and that of not taking a king from the other parcel likewiſe as 3 to 2, becauſe they being ſeparate parcels, the drawing a card from one, does not at all affect the drawing of one from the other, therefore the numbers continue the ſame in both probabilities, and are $\frac{2}{3}$ $\frac{2}{3}$, their products, give 4 and 9, and their difference 5 are the number of chances for taking a king from one of the parcels, and the probability as 9 to 5. The value of the expectation is thus found: in 3 guineas are 63 ſhillings; then multiply 63 by 5, and divide the product by 9, gives 35 ſhillings, the value required; ſo that if a perſon was to purchaſe my chance, he ought to give me one pound fifteen ſhillings for it.

Queſtion 14.

Suppoſe that out of a ſuit of 13 cards three cards be drawn, what are the odds that one of the three ſhall be an ace?

The

The probability that the first card drawn shall not be an ace is 13 to 12, or $\frac{12}{13}$; that of the second not being an ace as 12 to 11, or $\frac{11}{12}$; and that of the third not being an ace as 11 to 10, or $\frac{10}{11}$; and the total of these probabilities will be $\frac{12}{13}\ \frac{11}{12}\ \frac{10}{11}$, the product of the lower numbers is 1716, and that of the upper ones 1320; therefore the probability of the three cards being neither of them an ace is as 1716 to 1320; their difference 396 are the number of chances for one of the said three cards being an ace; so the odds are as 1716 to 396, or as 13 to 3.

Of throwing of Dice.

There being 6 faces on every die, the number of chances on any single die are consequently 6, and therefore it is as 6 to 1 that a person, with a single die, brings up an ace or any other particular face the first throw. The number of chances on 2 dice are 36, and are found by multiplying 6, the number of chances upon one die into itself; this multiplied by 6 again, produces 216, the number of chances on 3 dice, &c.

Question 15.

Required the probability of throwing an ace in one throw with two dice?

Solution. Imagine the ace excluded from each die, then there will be 5 faces upon each, and consequently

consequently all the different ways the two dice so reduced can come up, viz. 25 are the number of chances for missing an ace, which, subtracted from 36, all the chances upon two dice, leaves 11, the number of chances for throwing an ace in one throw, and the required probability as 36 to 11.

Question 16.

To find the probability of throwing an ace in one throw with three dice.

Solution. Imagine the ace excluded from each die, as before, then the throw will be made with three dice, each having but 5 faces, and the number of different ways, the three dice so reduced can come up, will be 5, multiplied by 5, and that product by 5 again, which makes 125; and this subtracted from 216, all the chances on three dice, leaves 91, the number of chances for throwing an ace in one throw, and the probability is as 216 to 91, or nearly as 12 to 5.

Question 17.

Required the probability of throwing two aces in one throw with four dice?

Solution. The probability of throwing an ace twice successively with one die is $\frac{1}{6}\ \frac{1}{6}$, and that of missing it twice $\frac{5}{6}\ \frac{5}{6}$; now collecting these together we have $\frac{1}{6}\ \frac{1}{6}\ \frac{5}{6}\ \frac{5}{6}$, and multiplying the upper numbers we have 25, and the lower ones 1,296; then

25 must be multiplied by as many different pairs as can be found in 4, that is 6, which will make 150, the number of chances for throwing two aces with four dice, and the probability as 1,296 to 150, or as 216 to 25.

Question 18.

Required the odds for throwing two aces in one throw with 6 dice?

Solution. The probability of throwing an ace twice successively with one die is $\frac{1}{6}\,\frac{1}{6}$, as before, and that of missing it four times $\frac{5}{6}\,\frac{5}{6}\,\frac{5}{6}\,\frac{5}{6}$; now collecting these together we have $\frac{1}{6}\,\frac{1}{6}\,\frac{5}{6}\,\frac{5}{6}\,\frac{5}{6}\,\frac{5}{6}$; multiply each set as before, and we have 625 and 46,656; now the number of different pairs in 6 is 15, which multiplied by 625, gives 9375; so the odds are as 46,656 to 9375, or nearly as 5 to 1.

Question 19.

A person engages, with a single die, to throw the ace first, the duce next, and so on, till he has thrown all the 6 faces regular one after the other, required the odds against him?

Solution. The odds are $\frac{1}{6}\,\frac{1}{5}\,\frac{1}{4}\,\frac{1}{3}\,\frac{1}{2}\,\frac{1}{1}$; the product of these 6 numbers being 720, therefore the odds are as 720 to 1.

Question

Question 20.

A person undertakes, in three casts with a common die, to throw both an ace and a duce, without regard to which shall happen to be first thrown; required his probability of winning.

Solution. The probability that he brings up an ace and a duce successively is $\frac{1}{6}\frac{1}{6}$, that is $\frac{1}{36}$, the probability that he misses an ace and a duce successively is $\frac{5}{6}\frac{4}{6}$, that is $\frac{20}{36}$; now both these probabilities together are $\frac{1}{36}\frac{20}{36}$, that is 1080 to 20; multiply 20 by the number of different pairs in 3, which is 3, and the product is 60; and this multiplied again by the number of throws, viz. 3, make 180, so the probability is as 1080 to 180, or as 6 to 1.

Any number of things given, as A, B, C, D, E, F, &c. to find the probability that in taking any number of them they shall be the A first, B next, &c.

Question 21.

Let there be a heap of 9 counters, marked A, B, C, &c. what is the probability that in taking three of them, they shall be the A first, B next, and C last?

Solution. The probability of taking A first is as 9 to 1, or $\frac{1}{9}$; if this happens, there remains 8

counters, and the probability of taking B next will be as 8 to 1, or $\frac{1}{8}$; we shall then have 7 counters left, and the C undrawn; and the probability of taking C next will be as 7 to 1, or $\frac{1}{7}$; now collecting all these together, we have $\frac{1}{9}\,\frac{1}{8}\,\frac{1}{7}$; the product of the lower number being 504, so the probability is as 504 to 1. If all the counters be required to be drawn, then the expression becomes $\frac{1}{9}\,\frac{1}{8}\,\frac{1}{7}\,\frac{1}{6}\,\frac{1}{5}\,\frac{1}{4}\,\frac{1}{3}\,\frac{1}{2}\,\frac{1}{1}$; all these numbers multiplied together produce 362,880, and are the number of different ways by which the said 9 counters can be drawn from the heap by one at a time, and consequently the number of chances against this event happening.

Question 22.

Let there be the same heap as before; what is the probability that in drawing three of them they shall be the same three first, marked A, B, C, as before, but without any regard to which shall be first drawn?

Solution. Consider the number of things to be drawn from the heap; place them above the line beginning with 1, 2, &c. likewise consider the whole number of things contained in the heap; begin with their number, which must be gradually decreased by one to the same number of places as are above the line, and place these below, which will form the expression for the probability of the happening of

of the event; then the expreſſion for the preſent queſtion will be $\frac{1}{9}\,\frac{2}{8}\,\frac{3}{7}$; now multiplying, as before taught, we have $\frac{6}{504}$; and ſeeing each of theſe numbers may be divided by 6, we reduce the expreſſion to $\frac{1}{84}$, or 84 to 1.

Queſtion 23.

Again let it be required to draw the four counters, A, B, C, D, at four drawings, without regard to which ſhall be firſt drawn, what is the probability of the ſucceſs?

Solution. The probability is expreſſed thus, $\frac{1}{9}\,\frac{2}{8}\,\frac{3}{7}\,\frac{4}{6}$; and multiplying as before, we have $\frac{24}{3024}$; now each of theſe may be divided by 6, which will reduce the expreſſion to $\frac{4}{504}$; this laſt may alſo be divided by 4, which reduces it to $\frac{1}{126}$, ſo the probability is as 126 to 1.

Let there be given a number of things, whereof ſome are ſeveral times repeated, for inſtance, A A, B B B, C C C C, &c. to find the probability, that by drawing theſe letters or things repreſented by them one by one, they ſhall come out in the order here placed; that is, that all the A's ſhall come out firſt, the B's next, &c.

Queſtion 24.

Suppoſe there are 9 counters, as before, but differently marked, for example: let two of them

be marked each A, three of them each B, and four of them each C; it is required to find the probability that drawing them one by one, all the A's shall come out first, the B's next, and the C's the last.

Solution. There being 9 counters in all, and two of them marked A, the probability of the A's coming out before either a B, or a C, will be $\frac{1}{9}\ \frac{2}{8}$; if this happens we have seven counters left, *viz.* three marked B and four marked C, and the probability of drawing the B's next is $\frac{1}{7}\ \frac{2}{6}\ \frac{3}{5}$; wherefore the probability of drawing the A's first, and the B's next, will be $\frac{1}{9}\ \frac{2}{8}\ \frac{1}{7}\ \frac{2}{6}\ \frac{3}{5}$, and multiplying, as before taught, we have $\frac{12}{15120}$, or by dividing each by 12, $\frac{1}{1260}$; so the probability of drawing the A's first, and the B's next, will be as 1260 to 1. If this happens, there is no need of proceeding any farther, for the taking the C's next is a certainty.

Question 25.

A certain person having 12 cards in his hand, of which two are hearts, three diamonds, three clubs, and the rest spades, desires another to draw them one at a time; what are the odds of that person's drawing the hearts first, and afterwards all the diamonds?

Solution. The probability of taking the hearts first is $\frac{1}{12}\ \frac{2}{11}$, and that of taking all the Diamonds next,

next, $\frac{1}{10}\ \frac{2}{9}\ \frac{3}{8}$; collecting these together we have $\frac{1}{12}\ \frac{2}{11}\ \frac{1}{10}\ \frac{2}{9}\ \frac{3}{8}$, and multiplying as before, we get $\frac{12}{95040}$, and dividing each part by 12, we reduce the expression to $\frac{1}{7920}$, whence the odds are as 7920 to 1.

Question 26.

Given the same as before, required the odds for taking the hearts and diamonds before either the clubs or spades, without regard to which sort shall be first taken?

Solution. Here being five cards to be drawn without regard to which shall be first, and 12 being the whole number of cards, the probability according to the rule given will be expressed thus $\frac{1}{12}\ \frac{2}{11}\ \frac{3}{10}\ \frac{4}{9}\ \frac{5}{8}$; now multiplying as before, we have $\frac{120}{950400}$, and taking away the cyphers in each, we get $\frac{12}{95040}$, and dividing both parts by 12, as before, we have $\frac{1}{7920}$, so the probability is as 792 to 1, which is 10 to 1 better than the former.

Question 27.

Given the same as before, required the probability of taking the clubs first, and spades next, before either of the hearts or diamonds?

Solution. There being 3 clubs and 4 spades, the probability of taking the three clubs first will be $\frac{1}{12}\ \frac{2}{11}\ \frac{3}{10}$; if this happens, we have 9 cards remaining, 4 of which being spades, must be drawn

next,

next, before either a heart or a diamond, and the probability of doing this will be thus expressed, $\frac{1}{9}\frac{2}{8}\frac{3}{7}\frac{4}{6}$; now collecting these two probabilities together, we get $\frac{1}{12}\frac{2}{11}\frac{3}{10}\frac{1}{9}\frac{2}{8}\frac{3}{7}\frac{4}{6}$; and multiplying as before taught, we have $\frac{144}{3991680}$; dividing each part by 12 we get $\frac{12}{332640}$, and dividing this again by 12, we have $\frac{1}{27720}$; so the probability is as 27720 to 1.

Question 28.

Given the same as before, required the probability of taking the three first sorts before either of the spades, without regard to which of the three sorts shall be first taken?

Solution. There being 4 spades, the numbers of cards constituting the other sorts will be 8 in all, and so many are required to be drawn, without regard to which sort comes out first; therefore the probability will be expressed thus, $\frac{1}{12}\frac{2}{11}\frac{3}{10}\frac{4}{9}\frac{5}{8}\frac{6}{7}\frac{7}{6}\frac{8}{5}$; now multiplying as before, we have $\frac{40320}{19958400}$ for the required probability, or $\frac{1}{495}$, that is, as 495 to 1.

To find the number of different ways by which two, three, or more things can be taken, varied, or differently disposed, in any certain number of those things given, which is commonly called the combination of quantities.

This problem extends to the several ways by which any number of things can be differently placed,

placed, so that every position may be different from the former. For example: I desire to know how many changes or different positions I can place the numbers 1, 2, 3, in, so that no two positions may be alike. The number of different positions will be 6, thus:

1	2	3
1	3	2
2	1	3
2	3	1
3	2	1
3	1	2

These are all the different ways by which three things can be placed; but if one more be added to them, the number of different positions in which 4 things may be ordered, will be increased to 24.

The solution of this problem is effected by multiplying the number of things to be varied or changed into each other, and the product will be the number of changes required. For example:

Question 29.

How many different positions can 6 persons place themselves in at a round table so that no two positions may be the same?

Solution. Multiply together the numbers 1, 2, 3, 4, 5, and 6, and their product 720, is the number

number of different positions required; so that if those 6 persons were to place themselves in a different position every day at dinner, it would be near two years before they would be found in that which they were in at first.

Question 30.

How many changes may be rung upon 12 bells?

Solution. Multiply together the numbers 1, 2, 3, &c. to 12, and their product is 479,001,600, which is the number of changes required; and if it were possible to ring 10 changes in a minute, it would be upwards of 91 years before the whole would be completed.

These I call simple combinations; the other part teaches the method of finding how many different ways two, three, or more things may be taken in any greater number of the same kind: For instance, I desire to know how many different pairs can be made with three duces? The answer is 3, and is thus proved; Suppose there are given the three following duces, marked 1, 2, 3, to find how many different pairs can be made out of them?

1	2	3
Two of Hearts.	Two of Clubs.	Two of Diam^{ds}.

1ſt. The firſt and ſecond make one pair.
2d. The firſt and third make one pair.
3d. The ſecond and third make one pair.
Which in the whole make three different pairs.

Again, let it be required how many different pairs can be made with 4 trois, the anſwer is 6, and is thus proved:

1	2	3	4
Three of Hearts.	Three of Clubs.	Three of Diam^{ds}.	Three of Spades.

1st. The first and second make one pair.
2d. The first and third make one pair.
3d. The first and fourth make one pair.
4th. The second and third make one pair.
5th. The second and fourth make one pair.
6th. The third and fourth make the sixth pair.

The manner of forming these respective pairs is well known to every cribbage-player, but the method of calculation none or few of them are acquainted with, though it is very simple, and performed as follows:

First, consider the whole number of cards or things to be paired, multiply their number by one less than the number given, and divide the product by 2, the quotient will be the number of different pairs to be found in that number: for example, take the 4 trois last given, multiply their number by one less, which is 3, and the product 12 divided by 2 gives 6, the number of different pairs required.

Let it be required to find how many different pairs can be formed with 6 aces.

Solution. Multiply 6, their number, by one less, which is 5, and the product 30, divided by 2, gives 15, the number of different pairs required.

When the number of things to be varied or taken exceed 2, proceed thus; begin with the whole number of them, and regularly decreasing them

them by one, place them one after the other, making as many places as there are things to be taken, then take the numbers 1, 2, 3, &c. to the ſame number of places, multiply each ſet into themſelves, and divide the greater product by the leſs, gives the number required.

Required how many different ways three things can be taken or combined in four?

Solution. The number of things to be taken are three; then beginning with 4, the whole number of things given, I place 4, 3, 2, to as many places as there are things to be taken, and beginning with 1, I make a regular increaſe to the ſame number of places, and I have the numbers 1, 2, 3; then multiplying each ſet into themſelves, I get 24 and 6, which being divided the one by the other, quotes 4, the number of different ways required.

Required the number of variations of 4 things in 6, or how many different ways I can chooſe 4 red balls out of 6, ſo that I may have a change every time?

Solution. The number of things to be taken being four, I begin with the whole number, of them, 6, and make four places thus, 6, 5, 4, 3, and alſo 1, 2, 3, 4; the product of the firſt four being 360, and that of the latter 24, I divide the one by the other, and the quotient gives 15, the anſwer required.

Here follows a Table of the number of variations of two, three, or more things in a greater number, as far as 12.

	3	4	5	6	7	8	9	10	11	12
Pairs,	3	6	10	15	21	28	36	45	55	66
Threes,	1	4	10	20	35	56	84	120	165	220
Fours,		1	5	15	35	70	126	210	330	495
Fives,			1	6	21	56	126	252	462	792
Sixes,				1	7	28	84	210	462	924
Sevens,					1	8	36	120	330	792
Eights,						1	9	45	165	495
Nines,							1	10	55	220
Tens,								1	11	66

The Uſe of the Table.

Suppoſe I deſire to know how many different ways five things may be taken five at a time out of nine, look for 9 at the head of the table, under which, and againſt five, is found 126, which are the number of different combinations of 5 things in 9.

Again, Suppoſe at the game of cribbage I have four duces, and deſire to know how many different pairs can be made out of them, I look for 4 at the head of the table, and againſt the word pairs I find 6, which

6, which are the number of different pairs required. Again, ſuppoſe at the game of cribbage I deſire to know how many fifteens I can make with 4 fives; thus, three fives making one fifteen, or what the cribbage-players call fifteen two, I look for the combinations of three things in four, viz. for 4 at the head of the table, and againſt the word three, I find 4, which ſhews there are four different ways of making fifteens with four fives, and gives, what is called at cribbage, fifteen eight. Thus may any hand at cribbage be made out from this table.

Queſtion 31.

Suppoſe there is a heap of 12 counters, conſiſting of four white and eight black, what is the probability that taking ſeven of them at random, three of the ſeven ſhall be white ones?

Solution, Conceive the 12 counters parted into two heaps, one containing four white and the other eight black; now, out of the four white counters three of them are required to be drawn, then ſeeking for the number of different ways by which three things can be taken out of four, which in the Table is found to be 4, and likewiſe for the number of different ways for taking four things out of eight, which are 70; then multiplying 4 by 70, we have 280, the number of different ways for taking three white counters and four black.

The

The whole number of different ways by which the seven counters can be taken out of twelve, or the number of combinations of seven things in twelve, are 792; therefore the probability of drawing three white counters and not more, is $\frac{280}{792}$; to which must be added the probability of taking four white and three black, which is thus found: the number of different ways for taking the four white counters can be but one, as there are but 4 of them; therefore this may be rejected; then seek for the number of different ways for taking three black out of eight black, which in the said Table is found to be 56, these chances added to the former, make 336, and the required probability is $\frac{336}{792}$, or $\frac{14}{33}$.

Question 32.

Three persons, A, B, and C, throw in their turns a certain ball, having four white faces and eight black ones, and he who shall first bring up a white one is to be reputed the winner; it is required to find the proportion of their respective probabilities for winning?

Solution. Find the probability of throwing a black one, which is $\frac{8}{12}$; multiply this by itself, that is, $\frac{8}{12}$ $\frac{8}{12}$, which gives $\frac{64}{144}$, then the proportion of their chances are 1, $\frac{8}{12}$ $\frac{64}{144}$; now to find these proportions in whole numbers, $\frac{8}{12}$ may be reduced to $\frac{2}{3}$, by dividing each part by 4, and $\frac{64}{144}$ may be reduced to $\frac{4}{9}$, by dividing each part by

by 16, then the proportions are reduced to 1 $\frac{2}{3}$ $\frac{4}{9}$; now $\frac{2}{3}$ is equal to $\frac{6}{9}$, then we have $\frac{6}{9}$ and $\frac{4}{9}$ in the same denomination for the proportion of the chances of the two latter players, the one 6 parts in 9, and the other 4 parts in 9; now the first players chance being an unit, contain the whole nine parts, therefore the proportions of their chances are 9, 6, and 4.

Question 33.

Four gamesters throw in their turns two dice each, with this condition, that he who shall bring up an ace first shall be reputed the winner; required their several probabilities of winning any sum proposed?

Solution. Find the probability of not throwing an ace the first time with two dice, which is $\frac{4}{6}$, that is $\frac{2}{3}$, multiply this by itself, and we get $\frac{4}{9}$, and this again by $\frac{2}{3}$, and the product is $\frac{8}{27}$; then the respective probabilities are 1 $\frac{2}{3}$ $\frac{4}{9}$ $\frac{8}{27}$; now reduce these expressions to an equality, thus, multiply $\frac{4}{9}$ by 3, makes $\frac{12}{27}$, and $\frac{2}{3}$ by 9, gives $\frac{18}{27}$, then we shall have $\frac{18}{27}$ $\frac{12}{27}$ and $\frac{8}{27}$, the proportion of the chances of the three last players; the lower number of each are the chances for the first, and the proportion of all their chances will be as 27, 18, 12, and 8.

Question

Queſtion 34.

Six people being engaged at a raffle, agree that each perſon ſhall throw one throw with three dice, and he who ſhall firſt bring up 12 faces ſhall be the winner; required their ſeveral probabilities of winning, and how much the odds are in favour of the firſt to that of the laſt?

Solution. The probability of throwing 12 faces with three dice is thus diſcovered: the number of chances on three dice are 216, as before proved; now if we have three dice thrown with 4 faces on each, we ſhall have what is required; therefore we have 3 chances on any ſingle die for throwing 4 faces, or a greater number, which will make 27 chances on 3 dice; but as there is the ſame chance for 12 faces to be thrown in any other form, and 4 faces on each ſingle die complete the buſineſs, multiply 27 by 3, the number of dice, and the product 81 gives the number of chances for throwing 12 faces with three dice, which ſubtracted from 216, the whole number of chances on three dice, gives 135, the number of chances againſt this event happening, and the probability $\frac{135}{216}$, that is, $\frac{5}{8}$; now $\frac{5}{8}$ $\frac{5}{8}$ gives $\frac{25}{64}$, $\frac{25}{64}$ $\frac{5}{8}$ gives $\frac{125}{512}$; $\frac{125}{512}$ $\frac{5}{8}$ gives $\frac{625}{4096}$; $\frac{625}{4096}$ $\frac{5}{8}$ gives $\frac{3125}{32768}$; therefore the proportion of their odds are 1, $\frac{5}{8}$, $\frac{25}{64}$, $\frac{125}{512}$, $\frac{625}{4096}$, $\frac{3125}{32768}$; thoſe reduced, give $\frac{20480}{32768}$, $\frac{10800}{32768}$, $\frac{8000}{32768}$, $\frac{5000}{32768}$, $\frac{3125}{32768}$, for the reſpective probabilities of the five laſt

last players; therefore the proportion of all their chances for winning will be as 32,768, 20,480, 10,800, 8000, 5000, and 3125; from whence we perceive that the first number is more than ten times the last, and consequently the first player has ten to one the best chance to that of the last.

Question 35.

A, B, and C, out of a heap of 12 counters, whereof four are white and eight black, draw one counter at a time, in this manner; A begins to draw first, B follows A, C follows B, then A begins again, and they continue to draw in the same order, till one of them, who is to be reputed the winner, draws the first white counter; what are their respective probabilities of winning any sum deposited?

Solution.

First, it is evident, that the play will be ended in nine drawings at most. Secondly, that the first, fourth, and seventh times of drawing will (should the play continue so long) belong to A; the second, fifth, and eighth to B; the third, sixth, and ninth to C. Thirdly, the probability of any one of the gamesters winning, when it is his turn to draw, is compounded of the probability of drawing a white counter at that time, and that of black ones being continually drawn before. The probability that A draws a white counter the first trial is $\frac{4}{12}$; but if this should fail, then the probability that he

fucceeds, when it is his turn to draw again, is $\frac{8}{12}\,\frac{7}{11}\,\frac{6}{10}\,\frac{4}{9}$, viz. that of three black ones being drawn fucceffively and a white one next. After the fame way of reafoning we get $\frac{8}{12}\,\frac{7}{11}\,\frac{6}{10}\,\frac{5}{9}\,\frac{4}{8}\,\frac{3}{7}\,\frac{4}{6}$, his probability of winning when he draws the laft time; whence A's total expectation is $\frac{4}{12}+\frac{8}{12}\,\frac{7}{11}\,\frac{6}{10}\,\frac{4}{9}+\frac{8}{12}\,\frac{7}{11}\,\frac{6}{10}\,\frac{5}{9}\,\frac{4}{8}\,\frac{3}{7}\,\frac{4}{6}$. B's probability of winning the firft trial is $\frac{8}{12}\,\frac{4}{11}$; that of his winning the fecond is $\frac{8}{12}\,\frac{7}{11}\,\frac{6}{10}\,\frac{5}{9}\,\frac{4}{8}$, and $\frac{8}{12}\,\frac{7}{11}\,\frac{6}{10}\,\frac{5}{9}\,\frac{4}{8}\,\frac{3}{7}\,\frac{2}{6}\,\frac{4}{5}$ is his probability for winning the third time of drawing, and hence his total expectation is equal to

$\frac{8}{12}\,\frac{4}{11}+\frac{8}{12}\,\frac{7}{11}\,\frac{6}{10}\,\frac{5}{9}\,\frac{4}{8}+\frac{8}{12}\,\frac{7}{11}\,\frac{6}{10}\,\frac{5}{9}\,\frac{4}{8}\,\frac{3}{7}\,\frac{2}{6}\,\frac{4}{5}$. Now as the third, fixth, and ninth times of drawing belong to C, his total expectation will be expreffed by $\frac{8}{12}\,\frac{7}{11}\,\frac{4}{10}+\frac{8}{12}\,\frac{7}{11}\,\frac{6}{10}\,\frac{5}{9}\,\frac{4}{8}\,\frac{4}{7}+\frac{8}{12}\,\frac{7}{11}\,\frac{6}{10}\,\frac{5}{9}\,\frac{4}{8}\,\frac{3}{7}\,\frac{2}{6}\,\frac{1}{5}\,\frac{4}{4}$; hence their refpective probabilities are reduced to $\frac{231}{495}$, $\frac{151}{495}$, $\frac{105}{495}$, or in the proportion of 77, 53, 35.

THE

THE SECOND PART.

THIS part contains the application of the former to the resolution of the several chances and probabilities in lotteries, upon purchasing and insuring tickets, or any other business relating thereto. Now, in the first place, in order to know the probabilities of success upon the purchasing of one or more tickets, and the odds against having any particular prize, the number of tickets and the number of those prizes must be considered. And here let it be observed, that the more numerous the number of tickets are, the greater will be the chances against having any particular prize: for if in a lottery of forty thousand tickets, there be two twenty thousand pound prizes, and in another of fifty thousand tickets, no more than two prizes of twenty thousand pounds, it is evident, the former is the most advantageous, as containing a less number of chances against the adventurer. But though the increasing the number of tickets affect the chances for having a capital prize, yet those for having a small prize, such as a twenty pound, remain nearly the same, there being generally two blanks to one prize; and therefore let the number of tickets be what they may,

the adventurer has ſtill an equal chance for one prize out of three tickets; but the odds are more than 100 to 1, that this prize exceeds a twenty pound. Seeing that the adventurer has an equal chance for having one prize in three tickets, it would follow that he has an equal chance for having two with double that number, &c. this may appear to be the caſe, but it is not ſtrictly true, for whenever the number of tickets are conſiderable, the number of chances in favour of the adventurer are increaſed according as the number of combinations are augmented by taking a greater number of things, ſo that in purchaſing 24 tickets it is more than an equal chance that 8 of thoſe tickets are prizes; this is mathematically demonſtrable, from whence we deduce a method of calculating how many tickets ſhould be purchaſed to make it an equal chance to have any particular prize; but the ſolution of this problem, depending upon a mathematical proceſs, will not come within the limits of this treatiſe, the following table is therefore given, by which may be ſeen how many tickets ſhould be purchaſed in a lottery of 50,000 tickets (which ſerves for the enſuing one) to make it an equal chance to have any one particular prize above a twenty pound.

A Table,

A Table, ſhewing the number of tickets that ſhould be purchaſed in the enſuing lottery, which begins drawing on the 12th of February, 1787, to make it an equal chance for the adventurer to have a prize above a 20*l.*

Prizes.	Number of Tickets.	Prizes.	Number of Tickets.
£. 20,000	16,724	£. 1000	1,576
10,000	11,149	500	1,153
5,000	6,841	100	346
2,000	3,460	50	139

The uſe of this table is evident, as, for example, ſuppoſe I deſire to know how many tickets ought to be purchaſed to make it an equal chance that I may have one prize of 500*l.* I look in the table for the number 500, under the word prizes, and againſt it I find 1153, the number of tickets required.

The principal engagements in the lottery conſiſt in the purchaſing tickets, and inſuring; the odds againſt the adventurer in the former depend partly on the price of tickets, and partly on the number of them in the wheel at the time of purchaſe; the latter depends but upon one cauſe, namely, the number of days drawing againſt the adventurer, and this is the moſt diſadvantageous of the two, which will be made to appear in what follows. To

enumerate

enumerate the ſeveral diſadvantages the adventurer labours under, and the odds he combats with in this laſt undertaking is the intent of the ſecond part of this treatiſe. And, firſt, the number of tickets, or number of days drawing againſt him, is not the only diſadvantage; in moſt kind of betts it is cuſtomary for the winning party to receive his own ſtake, and his adverſary's alſo; but the lottery-office-keeper acts quite contrary; if you lay him one guinea to twenty, or, which comes to the ſame thing, if you pay one guinea for inſurance, with a view to receive twenty, it is true, you receive twenty guineas in caſe you ſucceed; but in fact it is but nineteen: for the office-keeper retains the ſtake you have put down, and returns you but twenty guineas in the whole, whereas you ought, according to fair play, to receive one-and-twenty; for ſuppoſing the ſtakes depoſited in the hands of a third perſon, that perſon would certainly deliver the one-and-twenty guineas to the ſucceſsful party, and not think he had any right to retain one; conſequently the office-keeper ought to do the ſame: but he knows his intereſt too well to comply with ſuch a cuſtom, as he gains a profit of five per cent by this manœuvre; but it may be urged, that this is no more than a reaſonable profit. It is true, but he ſtops not here; another profit of five per cent ariſes from the calculation of the price of inſurance in his favour, but it

oftener

oftener amounts to ten or fifteen, and ſometimes to twenty per cent; ſo diſproportionate are they in general in the premium for inſuring, that I have known it, in the courſe of laſt lottery, to be upwards of thirty per cent in favour of the office-keeper, I mean on the premium for 20 guineas; for he has ſtill another profit accruing to him from the lower ſort of adventurers, who are generally the moſt numerous; this ariſes from the diviſion of the price of inſurance for twenty guineas into halves, quarters, &c. for whenever this happens to be a ſum which cannot be equally made into thoſe ſeveral diviſions without fractions, the farther thoſe diviſions are continued, the greater will be the diſproportion. For example, ſuppoſe the premium for inſuring for 20 guineas be *9s. 6d.* then for the half or 10 guineas it will be 4*s.* *9d.* for a quarter, or 5 guineas, it ought to be 2*s.* 4½*d.* but the office-keeper, who ſcorns to admit of halfpence or farthings into his accounts, makes the quarter 2*s.* 5*d.* the eighth, which is the half of this, he will make 1*s.* *3d.* and a ſixteenth will be 8*d.* now, 16 eightpences make 10*s.* 8*d.* for the premium for 20 guineas, which is more than the former by 14*d.* this makes another profit of more than 12 per cent; ſo that upon the whole the office-keeper clears, upon an average, from 35 to 40 per cent, and muſt certainly have the advantage of you in every undertaking of this kind; and if you ſhould be ſo

ſucceſsful

ſucceſsful as to ſet down a gainer at the concluſion of your ſport, it muſt be by mere good fortune, and not in conſequence of any odds you can poſſibly have in your favour. And now let me give a word of advice to the inſurer of tickets or numbers. In the firſt place I muſt warn him againſt taking too many numbers at once; this is what ruins moſt adventurers, who think, that by taking a long ſtring of numbers, they make it almoſt a certainty of having one every night; not conſidering that, if they chance to miſs two, three, or four nights together, the expence of a ſtring of thirty or forty numbers will ſoon diminiſh their former gains, if any; my advice is, therefore, not to take more than ſix numbers at a time for the moſt, and I would rather chooſe three or four; for the odds for your gain, at the beginning of the lottery eſpecially, will be nearly the ſame. The next thing I would adviſe him is, not to keep on any number or numbers too long a time, but to change them for others: becauſe there is equally the ſame chance for any other number to come up that he has not had before, as there is for any one of thoſe to riſe that he may have inſured, perhaps for ſeveral days, without any ſucceſs. Thirdly and laſtly, I muſt declare my ſentiments on the mode of inſuring following numbers; this I look upon to be the worſt of any, and that in which the adventurer will be ſure to loſe, if he perſiſts in it; as I have ſhewn

in the First Part of this Treatise, the very great probability there is against any certain number of events happening successively; for I see no other view there can be in taking following numbers, than a supposition that two or three of them may be drawn in one day; for if this is not the adventurer's expectation, he may as well take any other numbers: but the probability of two or three following numbers being drawn in one day, except towards the conclusion of the lottery, is amazingly great, and it is, in my opinion, next to madness to proceed upon this mode of insuring. I grant so far as this, that there is the very same chance for one of such numbers to be drawn, as there is for any other, but not for more than one; and I will give this farther advice, that whenever it happens that you have a number drawn out of a string of following ones, be sure you drop the whole of them the next day, for the odds against you, that another shall be drawn the following day, are very great indeed, except, as I observed before, towards the conclusion of the lottery.

In order to find at any time the probability of your success upon insuring any certain number of tickets, the first thing to be considered is, the number of days unexpired of the drawing, and then the number of tickets insured, and the mode of insuring, whether for blank and prize, or in any other accustomed manner.

Example 1.

Suppose I insure three numbers, when there are 17 days to draw, what is the probability that I shall have one of them drawn that day?

Solution. The number of days being 17 to finish the drawing, one of which will be completed the same day I insure, therefore I have 16 days against me, and 3 numbers for me, and the probability that I shall have one of these numbers that same day is as 16 to 3; now to find the value of my expectation, suppose these numbers are insured for 20 guineas each, or 21 pounds, then the value of my expectation will be found by multiplying 21*l.* by 3, which gives 63, and dividing this by 16, makes 3*l.* 18*s.* 9*d.* and this is the sum I ought to give for my insurance, exclusive of the allowance the office-keeper takes, called his commission or discount.

Example 2.

Let it be required to find the probability of success upon the above numbers, if insured for blanks only.

The probability of one of the numbers drawn being either a blank or a prize was found to be $\frac{3}{16}$; now this is the probability of the number drawn being either the one or the other, no matter which; but I desire to know the probability

of

of the ſaid drawn number being abſolutely a blank, I conſider, that as there are nearly two blanks to a prize, the proportion of the whole number of tickets contained in the lottery will be to the blanks as 3 to 2; therefore the former probability $\frac{3}{16}$ multiplied by $\frac{2}{3}$ will give the probability that the number drawn may be a blank, which is expreſſed thus, $\frac{3}{16}\frac{2}{3}$; then multiplying 2 by 3, I have 6, and 16 by 3 I have 48, therefore the probability is $\frac{6}{48}$ or $\frac{1}{8}$, and the odds as 8 to 1; and the value of the inſurance will in this caſe be the eighth part of 20 guineas, or £2 : 12 : 6.

If it be required to find the probability of the ſaid drawn number being a prize, I conſider the proportion of the prizes to the whole number of tickets, which is one third, for the blanks from the former example appear to make two third parts of the whole number of tickets, therefore the prizes conſtitute the other third, and the probability of the ſaid number being drawn a prize will be three times that of its being drawn a blank or a prize, or twice that of its being drawn a blank. Now the odds for its being either a blank or a prize were found to be as 16 to 3, then multiplying 16 by 3, gives 48, and the odds are as 48 to 3, or as 16 to 1, and the value of the inſurance is half the former, viz. £1 ; 6 : 3.

Example 3.

Suppoſe 5 numbers to be inſured when there are 13 days to complete the drawing, what are the odds that two of them ſhall be drawn on that day either blanks or prizes?

Solution. Here being 13 days to draw, I have 12 days againſt me, and 5 numbers for me, and the odds for one of them being drawn that day either a blank or a prize are as 12 to 5; ſuppoſe this to happen, then there are 4 numbers left for me, and ſtill the ſame number of days againſt me, therefore the odds for having a ſecond drawn are as 12 to 4, and the probability of both theſe events happening is $\frac{5}{12}$ $\frac{4}{12}$, then multiplying as before taught, I have $\frac{20}{144}$, the probability required; divide both parts by 4, and the expreſſion is reduced to $\frac{5}{36}$, ſo the odds againſt having two of the five numbers drawn that day are as 36 to 5, or better than 7 to 1.

If it be required to find the odds of both the numbers being drawn blanks, I proceed thus, the probability of one of them being drawn a blank is $\frac{5}{12}$ $\frac{2}{3}$, which by multiplying give $\frac{10}{36}$, and that of another being drawn a blank $\frac{4}{12}$ $\frac{2}{3}$, which give $\frac{8}{36}$, and the probability of both theſe events happening will be $\frac{10}{36}$ $\frac{8}{36}$, then multiplying 10 by 8 I get 80, and 36 by 36, 1296, or $\frac{80}{1296}$, and dividing both parts by 8, I have $\frac{10}{162}$, and this again by 2 gives

gives $\frac{5}{81}$, ſo the odds are as 81 to 5, or about 16 to 1.

If it be required to find the probability of both the numbers drawn, being prizes, I proceed thus, the probability of one of them being drawn a prize is $\frac{5}{12}\,\frac{2}{3}$, that is, $\frac{10}{36}$, and that of another being ſo drawn a prize $\frac{6}{12}\,\frac{2}{3}$, that is, $\frac{12}{36}$, likewiſe that of both theſe happening $\frac{10}{36}\,\frac{12}{36}$, then multiplying as uſual, I have $\frac{120}{1296}$ for the required probability, or nearly as 130 to 2.

The odds that one may be drawn a blank and the other a prize are nearly the ſame as the firſt, namely, that of their being either blanks or prizes.

Example 4.

Suppoſe I have three numbers inſured when there are five days to draw, what are the odds that two of them may be drawn that day?

Solution. The probability that one of them may be drawn is $\frac{3}{4}$, and that of another being ſo $\frac{2}{4}$, and the expreſſion for both theſe events happening will be $\frac{3}{4}\,\frac{2}{4}$, and multiplying as before, I have $\frac{6}{16}$ or $\frac{3}{8}$, ſo the odds are as 8 to 3.

To find the probability that both the numbers drawn may be blanks I proceed thus: the probability that one of them ſhall be a blank is $\frac{3}{4}\,\frac{2}{3}$, that is, $\frac{6}{12}$, or as 2 to 1, the probability that another may

be

be a blank is $\frac{2}{4}\ \frac{2}{3}$, that is, $\frac{4}{12}$ or $\frac{1}{3}$; therefore the probability of both these events happening is $\frac{1}{2}\ \frac{1}{3}$, that is, $\frac{1}{6}$, and the odds as 6 to 1.

If it be required to find the probability of the two numbers drawn being prizes, I find the probability of the first being drawn a prize, which is $\frac{3}{4}\ \frac{1}{3}$, that is, $\frac{3}{12}$, or $\frac{1}{4}$; the probability of the next being a prize is $\frac{2}{4}\ \frac{1}{3}$, that is, $\frac{2}{12}$, or $\frac{1}{6}$; and that of both these events happening $\frac{1}{4}\ \frac{1}{6}$, which gives $\frac{1}{24}$, and the odds as 24 to 1.

Example 5.

I have 3 numbers insured for blank and prize, on the 28th day of drawing, in a lottery of 50,000 tickets, what are the odds that all the three shall be drawn either blanks or prizes on that same day?

Solution. I consider the whole number of days for the drawing of a lottery of 50,000 tickets, which is seldom less than 37, then there will be 9 days to come, exclusive of the day I insure; therefore the probability of having all the three numbers drawn that day will be thus expressed, $\frac{3}{9}\ \frac{2}{9}\ \frac{1}{9}$; the product of the lower numbers is 729, and that of the upper ones 6, which make $\frac{6}{729}$, and dividing each part by 3, I have $\frac{2}{243}$ for the required probability, and the odds about 122 to 1.

To

To find the probability that two out of the three shall be prizes.

The probability of the first being a prize is $\frac{3}{9}$ $\frac{1}{3}$, that is, $\frac{3}{27}$, or $\frac{1}{9}$; and that of the second being a prize $\frac{2}{9}$ $\frac{1}{3}$, that is, $\frac{2}{27}$; and the probability of the remaining one being a blank is $\frac{1}{9}$ $\frac{2}{3}$, that is, $\frac{2}{27}$; now collecting all these together, I have $\frac{1}{9}$ $\frac{2}{27}$ $\frac{2}{27}$; and multiplying, as before, $\frac{4}{6561}$; but as the first number drawn may happen to be a blank, or the second may happen to be the same, and as there is the same chance for the three numbers to be drawn in any other form, it follows, that as many different ways as two things can be taken out of three, just so many times must this probability be repeated to give the required one; then looking in the Table of Combinations, in the First Part, for the number of variations of two things in three I find 3, and multiplying the upper number 4 by 3, I have 12, so the true probability is $\frac{12}{6561}$, and the odds as 6561 to 12, or nearly as 547 to 1.

Example 6.

Let there be four numbers taken when there are six days to draw, required the probability that all of them may be drawn that day either blanks or prizes; likewise the probability that two of the four shall be prizes?

Solution.

Solution. Here being 6 days to draw, one of which will be completed before my lot is determined, I have therefore 5 days against me, and 4 numbers for me, and the probability of having one single number will be $\frac{4}{5}$, and the odds as 5 to 4; then the probability of having all the four drawn will be $\frac{4}{5}\,\frac{3}{5}\,\frac{2}{5}\,\frac{1}{5}$, and multiplying, as before, I have $\frac{24}{625}$, which is nearly as 26 to 1.

To find the probability that two of the four shall be prizes proceed thus. The probability of the first number being drawn a prize will be $\frac{4}{5}\,\frac{1}{3}$, that is, $\frac{4}{15}$; and that of the second being a prize will be $\frac{3}{5}\,\frac{1}{3}$, that is, $\frac{3}{15}$; the probability of the other two being blanks will be $\frac{2}{5}\,\frac{2}{3}$ and $\frac{1}{5}\,\frac{2}{3}$, that is, $\frac{4}{15}$ and $\frac{2}{15}$; now collecting all these together we have $\frac{4}{15}\,\frac{3}{15}\,\frac{4}{15}\,\frac{2}{15}$, their products give $\frac{96}{50625}$; but as there is an equal chance for any two of the four numbers to be prizes, this probability must be repeated as many times as two things can be taken or combined in four, which, by the Table is 6; then multiplying 96 by 6, gives 576, so the true probability is $\frac{576}{50625}$, which is about 88 to 1.

Example 7.

Let there be 12 numbers taken when there are 26 days to draw, what is the probability that three of them may be drawn that day, and that two of the three shall be prizes?

Solution.

Solution. The probability that three of thoſe numbers ſhall be drawn that day either blanks or prizes is thus expreſſed, $\frac{12}{25}\,\frac{11}{25}\,\frac{10}{25}$, that is $\frac{1320}{15625}$, and the odds nearly as 12 to 1.

Now to find the probability that two of the three numbers drawn ſhall be prizes. The probability that the firſt ſhall be drawn a prize is $\frac{12}{25}\,\frac{1}{3}$, that is, $\frac{12}{75}$, or $\frac{4}{25}$; the probability that the next may be a prize is $\frac{11}{25}\,\frac{1}{3}$, that is, $\frac{11}{75}$; and the probability of the third being a blank is $\frac{10}{25}\,\frac{2}{3}$, that is, $\frac{20}{75}$, or $\frac{4}{15}$; now collecting all theſe together, we have $\frac{4}{25}\,\frac{11}{75}\,\frac{4}{15}$, and multiplying, as before, we get $\frac{176}{28125}$; but as there is the very ſame chance for any two of the three drawn numbers being prizes, I multiply, as before, by the number of combinations of two things in three, which are 3, and the product gives 528; therefore the true probability is $\frac{528}{28125}$, and the odds nearly as 53 to 1.

Example 8.

Suppoſe 24 numbers to be taken when there are 21 days to draw, what is the probability of having two of theſe numbers drawn, and that one of the two ſhall be a prize?

Solution. In order to ſolve this queſtion, I find the probability of the two numbers drawn being blanks, and ſubtracting that from the whole number of chances, the remainder will give the number of chances for one of the two numbers being a

prize; thus the probability of the first number being drawn a blank is $\frac{24}{30}$ $\frac{2}{2}$, that is, $\frac{48}{60}$, or $\frac{4}{5}$; that of the next being a blank $\frac{23}{30}$, $\frac{2}{2}$, that is, $\frac{46}{60}$; and that of both these events happening $\frac{4}{5}$ $\frac{46}{60}$, that is, $\frac{184}{300}$; now taking 184 from 300, there remains 116; therefore the required probability is $\frac{116}{300}$, and the odds as 300 to 116.

Example 9.

I have 3 numbers insured, when there are 5 days to draw, for prize only, what is the probability that I shall have one of these numbers drawn as insured, that is, a prize?

Solution. Find the probability of the 3 numbers being drawn blanks, and subtracting that from the whole number of chances, the remainder will be the chances for having one prize. Thus the probability of the three numbers being drawn blanks will be $\frac{3}{4}$ $\frac{2}{3}$ $\frac{2}{4}$ $\frac{2}{3}$ $\frac{1}{4}$ $\frac{2}{3}$; then multiplying, as before, we have $\frac{48}{1728}$, and subtracting 48 from 1728, there remains 1680; therefore the probability is $\frac{1680}{1728}$, or as 36 to 35.

Example 10.

Let there be two sets of numbers of 6 in a set, one set being insured for blanks and prizes, and the other for prize only, what are the odds for having one of those numbers drawn out of the two sets, and likewise the odds for having one drawn

drawn out of each, supposing there are 11 days to draw?

Solution. The probability of one being drawn out of the first set is $\frac{6}{30}$, and that of one being drawn out of the other $\frac{6}{30}\frac{3}{1}$, that is, $\frac{18}{30}$, or $\frac{3}{5}$; now the first probability $\frac{6}{30}$, may be reduced to $\frac{1}{5}$, so we have one chance in five for the one, and 3 in 5 for the other; therefore the odds are as 5 to 4.

Now for the latter part of the question, the probability of one being drawn in the first set is $\frac{6}{10}$, and that of one being drawn in the other set is $\frac{6}{10}\frac{1}{3}$, or $\frac{6}{30}$, equal to $\frac{1}{5}$; and the probability of both these events happening will be $\frac{6}{10}\frac{1}{5}$, that is, $\frac{6}{50}$; both parts of which being divided by 2, give $\frac{3}{25}$, so the odds for having one of each set are as 25 to 3.

Example 11.

Suppose I have two sets of numbers, one containing 4, and the other 6, the first being insured for blank and prize, and the other for prize only; required the probability of having one number drawn out of the two sets, also the probability of having one drawn in each, supposing there are 9 days to draw.

Solution. The probability of one of the first set being drawn will be $\frac{4}{8}$, and that of one of the second set being drawn $\frac{6}{8}\frac{1}{3}$, that is, $\frac{6}{24}$; now di-

 viding

viding each of these parts, 6 and 24, by 3, we reduce the expression to $\frac{2}{8}$, therefore we have 2 chances in 8 to win in this set, and 4 chances in 8 in the other, which in the whole make 6 chances in 8, so the odds are as 8 to 6, or as 4 to 3; now to find the probability of having one drawn in each set, the probability of one of the first set being drawn was found to be $\frac{4}{8}$, and that of one being drawn in the other $\frac{2}{8}$, then the probability of both these events happening will be $\frac{4}{8}$ $\frac{2}{8}$, that is, $\frac{8}{64}$, or $\frac{1}{8}$, and the odds as 8 to 1.

Example 12.

Let there be 3 sets of numbers, 6 in a set, taken when there are 9 days to draw, the first insured for blank and prize, the second for blank only, and the third for prize only, what is the probability that I shall have one out of the whole?

Solution. The probability that I may have one of the first set is $\frac{6}{8}$; that of one of the second $\frac{6}{8}$ $\frac{2}{3}$, that is, $\frac{12}{24}$; and that of having one of the third $\frac{6}{8}$ $\frac{1}{3}$, that is, $\frac{6}{24}$; now I reduce the two latter chances to the same denomination as the first, thus, $\frac{12}{24}$ divided by 3, gives $\frac{4}{8}$, and $\frac{6}{24}$ divided by 3, gives $\frac{2}{8}$; then I perceive that the chances against me are 8, and those in my favour are 6, 4 and 2, whose sum is 12, therefore the odds are as 8 to 12, or as 2 to 3.

Example

Example 13.

Suppoſe three numbers to be taken, when there are 11 days to draw, what are the odds that one of the three ſhall be drawn on the third time of inſuring them, either a blank or a prize?

Solution. The number of chances againſt any one number being drawn the firſt day they are inſured is 10, and the chances againſt this on the two following days are 9 and 8; now multiply theſe three chances together, the product is 720, the whole number of chances againſt the event happening; divide this number of chances, 720, by thoſe of the firſt day, which are 10, and the quotient is 72. The number of chances for one number being drawn the firſt day are 3; multiply this by 72, the ſaid quotient, the product is 216, and this again by the number of trials for the event, that is, 3 days, the product is 648, the whole number of chances for the event to happen; now the number of chances againſt it was found to be 720, therefore the odds are as 720 to 648, or as 10 to 9, being nearly an equal chance.

Example 14.

Let there be 4 numbers taken, when there are 10 days to draw, what are the odds that one of theſe numbers may be drawn in 4 days?

Solution.

Solution. There being 10 days to draw, the number of chances against one number being drawn the first day is 9, and those of the three following days 8, 7, 6; then multiplying these four chances together, we have 3024, the whole number of chances against the event happening; divide these chances by those of the first day, which are 9, and we have 336. Now the number of chances for any one number being drawn the first day are 4, multiply this by 336, we have 1344, and this again by 4, the number of days, and the product is 5376, the number of chances for this event to happen; now the number of chances against it was found to be 3024, therefore the odds are as 3024 to 5376, or about 9 to 16.

Example 15.

Let there be 15 numbers insured for blank and prize, on the first day of drawing, in a lottery of 50,000 tickets, what are the odds that one of them shall be drawn in three days?

Solution. Allowing 37 days for the whole time of drawing, we have 36 chances against us on the first day, and those of the two succeeding ones will be 35, 34; multiplying these three chances together, we have 42,840, and dividing this by the number of chances of the first day, viz. 36, we have 1190. Now the number of chances for any one number bring drawn the first day is 15,

which

which multiplied by 1190, gives 17,850; then multiplying this last product by 3, the number of days given in the question, we have 53,550, therefore the odds are as 42,840 to 53,550, or as 68 to 85.

A Table, shewing what the Premium for insuring Tickets ought to be for every Day in the ensuing Lottery, which begins drawing February 12, 1787.

For twenty Guineas.																			
Days.	B. & P.			B. only.			P. only.			Days.	B. & P.			B. only.			P. only.		
	l.	s.	d.	l.	s.	d.	l.	s.	d.		l.	s.	d.	l.	s.	d.	l.	s.	d.
1	0	12	0	0	8	0	0	4	0	19	1	3	0	0	15	6	0	7	9
2	0	12	0	0	8	0	0	4	0	20	1	4	6	0	16	3	0	8	2
3	0	12	6	0	8	6	0	4	3	21	1	6	0	0	17	6	0	8	9
4	0	13	0	0	8	6	0	4	3	22	1	7	6	0	18	6	0	9	3
5	0	13	0	0	8	6	0	4	3	23	1	9	0	0	19	6	0	9	9
6	0	13	6	0	9	0	0	4	6	24	1	11	6	1	1	0	0	10	6
7	0	14	0	0	9	6	0	4	9	25	1	14	0	1	2	9	0	11	4
8	0	14	6	0	9	9	0	4	10	26	1	16	9	1	4	6	0	12	3
9	0	15	0	0	10	0	0	5	0	27	2	0	0	1	6	9	0	13	6
10	0	15	9	0	10	6	0	5	3	28	2	4	0	1	9	6	0	14	9
11	0	16	6	0	11	0	0	5	6	29	2	9	0	1	12	9	0	16	6
12	0	17	0	0	11	6	0	5	9	30	2	15	0	1	16	9	0	18	6
13	0	17	6	0	11	9	0	5	10	31	3	3	0	2	2	0	1	1	0
14	0	18	4	0	12	3	0	6	2	32	3	13	6	2	9	0	1	4	6
15	0	19	0	0	12	9	0	6	4	33	4	8	0	2	18	9	1	9	6
16	1	0	0	0	13	6	0	6	9	34	5	10	0	3	13	6	1	16	9
17	1	1	0	0	14	0	0	7	0	35	7	7	0	4	18	0	2	9	0
18	1	2	0	0	14	9	0	7	4	36	11	0	0	7	6	9	3	13	6

For ten Guineas.

Days.	B. & P. l.	s.	d.	B. only. l.	s.	d.	P. only. l.	s.	d.	Days.	B. & P. l.	s.	d.	B. only. l.	s.	d.	P. only. l.	s.	d.
1	0	6	0	0	4	0	0	2	0	19	0	11	6	0	7	9	0	3	11
2	0	6	0	0	4	0	0	2	0	20	0	12	3	0	8	2	0	4	1
3	0	6	3	0	4	3	0	2	2	21	0	13	0	0	8	9	0	4	5
4	0	6	6	0	4	3	0	2	2	22	0	13	9	0	9	3	0	4	8
5	0	6	6	0	4	3	0	2	2	23	0	14	6	0	9	9	0	4	11
6	0	6	9	0	4	6	0	2	3	24	0	15	9	0	10	6	0	5	3
7	0	7	0	0	4	9	0	2	5	25	0	17	0	0	11	5	0	5	8
8	0	7	3	0	4	11	0	2	5	26	0	18	5	0	12	3	0	6	2
9	0	7	6	0	5	0	0	2	6	27	1	0	0	0	13	5	0	6	9
10	0	7	11	0	5	3	0	2	8	28	1	2	0	0	14	9	0	7	5
11	0	8	3	0	5	6	0	2	9	29	1	4	6	0	16	5	0	8	3
12	0	8	6	0	5	9	0	2	11	30	1	7	6	0	18	5	0	9	3
13	0	8	9	0	5	11	0	2	11	31	1	11	6	1	1	0	0	10	6
14	0	9	2	0	6	2	0	3	1	32	1	16	9	1	4	6	0	12	3
15	0	9	6	0	6	5	0	3	2	33	2	4	0	1	9	5	0	14	9
16	0	10	0	0	6	9	0	3	5	34	2	15	0	1	16	9	0	18	5
17	0	10	6	0	7	0	0	3	6	35	3	13	6	2	9	0	1	4	6
18	0	11	0	0	7	5	0	3	8	36	5	10	0	3	13	5	1	16	9

For five Guineas.

Days.	B. & P. l.	s.	d.	B. only. l.	s.	d.	P. only. l.	s.	d.	Days.	B. & P. l.	s.	d.	B. only. l.	s.	d.	P. only. l.	s.	d.
1	0	3	0	0	2	0	0	1	0	19	0	5	9	0	3	11	0	2	0
2	0	3	0	0	2	0	0	1	0	20	0	6	2	0	4	1	0	2	1
3	0	3	2	0	2	2	0	1	1	21	0	6	6	0	4	5	0	2	3
4	0	3	3	0	2	2	0	1	1	22	0	6	11	0	4	8	0	2	4
5	0	3	3	0	2	2	0	1	1	23	0	7	3	0	4	11	0	2	6
6	0	3	5	0	2	3	0	1	1	24	0	7	11	0	5	3	0	2	8
7	0	3	6	0	2	5	0	1	3	25	0	8	6	0	5	9	0	2	10
8	0	3	8	0	2	6	0	1	3	26	0	9	3	0	6	2	0	3	1
9	0	3	9	0	2	6	0	1	3	27	0	10	0	0	6	9	0	3	5
10	0	4	0	0	2	8	0	1	4	28	0	11	0	0	7	5	0	3	9
11	0	4	2	0	2	9	0	1	4	29	0	12	3	0	8	3	0	4	2
12	0	4	3	0	2	11	0	1	6	30	0	13	9	0	9	3	0	4	8
13	0	4	5	0	3	0	0	1	6	31	0	15	9	0	10	6	0	5	3
14	0	4	7	0	3	1	0	1	7	32	0	18	5	0	12	3	0	6	2
15	0	4	9	0	3	3	0	1	7	33	1	2	0	0	14	9	0	7	5
16	0	5	0	0	3	5	0	1	9	34	1	7	6	0	18	5	0	9	3
17	0	5	3	0	3	6	0	1	9	35	1	16	9	1	4	6	0	12	3
18	0	5	6	0	3	9	0	1	10	36	2	15	0	1	16	9	0	18	5

For two Guineas and a Half.

Days.	B. & P. l.	B. & P. s.	B. & P. d.	B. only. l.	B. only. s.	B. only. d.	P. only. l.	P. only. s.	P. only. d.	Days.	B. & P. l.	B. & P. s.	B. & P. d.	B. only. l.	B. only. s.	B. only. d.	P. only. l.	P. only. s.	P. only. d.
1	0	1	6	0	1	0	0	0	6	19	0	2	11	0	2	0	0	1	0
2	0	1	6	0	1	0	0	0	6	20	0	3	1	0	2	1	0	1	1
3	0	1	7	0	1	1	0	0	7	21	0	3	3	0	2	3	0	1	2
4	0	1	8	0	1	1	0	0	7	22	0	3	6	0	2	4	0	1	2
5	0	1	8	0	1	1	0	0	7	23	0	3	8	0	2	6	0	1	3
6	0	1	9	0	1	2	0	0	7	24	0	4	0	0	2	8	0	1	4
7	0	1	9	0	1	3	0	0	8	25	0	4	3	0	2	11	0	1	5
8	0	1	10	0	1	3	0	0	8	26	0	4	8	0	3	1	0	1	7
9	0	1	11	0	1	3	0	0	8	27	0	5	0	0	3	5	0	1	9
10	0	2	0	0	1	4	0	0	8	28	0	5	6	0	3	9	0	1	11
11	0	2	1	0	1	5	0	0	9	29	0	6	2	0	4	2	0	2	1
12	0	2	2	0	1	6	0	0	9	30	0	6	11	0	4	8	0	2	4
13	0	2	3	0	1	6	0	0	9	31	0	7	11	0	5	3	0	2	3
14	0	2	4	0	1	7	0	0	10	32	0	9	3	0	6	2	0	3	1
15	0	2	5	0	1	8	0	0	10	33	0	11	0	0	7	5	0	3	5
16	0	2	6	0	1	9	0	0	11	34	0	13	9	0	9	3	0	4	8
17	0	2	8	0	1	9	0	0	11	35	0	18	5	0	12	3	0	6	2
18	0	2	9	0	1	11	0	0	11	36	1	7	6	0	18	5	0	9	3

One Guinea. B. & P.

Days.	s.	d.	Days.	s.	d.
1	0	7	19	1	2
2	0	7	20	1	3
3	0	7	21	1	4
4	0	8	22	1	5
5	0	8	23	1	6
6	0	8	24	1	7
7	0	9	25	1	8
8	0	9	26	1	10
9	0	9	27	2	0
10	0	9	28	2	2
11	0	10	29	2	5
12	0	10	30	2	9
13	0	10	31	3	2
14	0	11	32	3	8
15	0	11	33	4	5
16	1	0	34	5	6
17	1	1	35	7	4
18	1	1	36	11	0

THE THIRD PART.

THIS part contains thirty games of draughts, selected from the manner of playing practised by some of the best players at the present time. The difficulty of playing this game with propriety is incontestible; for among the multitude that practise it very few understand it. There are indeed not many who by any frequency of playing can attain a moderate degree of skill without examples and instructions: it is for this reason that we have here given a collection of the most artful games, the most critical situations, and the most striking revolutions, which can possibly occur.

An Explanation of the following Games.

I. Place the draught-board with what the players call the double-corner towards the right hand.

II. The board being placed in this manner, the white squares are numbered in order from 1 to 32.

III. Observe that the black pieces are placed upon the first twelve squares in all the following games.

IV. For the playing of any move required, the numbers may be written upon the board itself, near a corner of each square, so as to be easily seen when the men are placed; or a table may be drawn upon paper or card, and the squares numbered as in

in the following figure, and ſuch a table will be ready to guide to any move directed.

V. Then to play any of the following games you muſt move from one ſquare to the other as marked in the game you mean to play. For inſtance, in the firſt game, the firſt move is a black man, which is from 11 to 15; the ſecond move being a white man, is from 22 to 17; the third move is a black man from 8 to 11, &c. Thus may any move be eaſily made.

THE DRAUGHT BOARD.

	1		2		3		4
5		6		7		8	
	9		10		11		12
13		14		15		16	
	17		18		19		20
21		22		23		24	
	25		26		27		28
29		30		31		32	

Game 1.

B. from	11 to	15	W.	22	17	B.	8	11	W.	25	22
B.	9	13	W.	23	18	B.	6	9	W.	27	23
B.	9	14	W.	18	9	B.	5	14	W.	30	25
B.	1	6	W.	24	19	B.	15	24	W.	28	19
B.	11	15	W.	32	28	B.	15	24	W.	28	19
B.	7	11	W.	22	18	B.	13	22	W.	18	9
B.	6	13	W.	25	18	B.	3	8	W.	18	14
B.	10	17	W.	21	14	B.	11	16	W.	14	9
B.	2	7	W.	9	6	B.	7	10,	&c,		

Drawn Game.

Game 2.

B. from	11 to	15	W.	22	17	B.	8	11	W.	25	22
B.	11	16	W.	23	18	B.	3	8	W.	18	11
B.	8	15	W.	24	19	B.	15	24	W.	27	11
B.	7	16	W.	22	18	B.	9	14	W.	18	9
B.	5	14	W.	28	24	B.	4	8	W.	24	19
B.	16	23	W.	26	19	B.	8	11	W.	31	26
B.	2	7	W.	26	23	B.	11	15	W.	32	28
B.	15	24	W.	28	19	B.	7	11	W.	30	26
B.	11	15	W.	19	16	B.	12	19,	&c.		

Drawn Game.

Game 3.

B. from	11 to	15	W.	22	17	B.	8	11	W.	25	22
B.	11	16	W.	23	18	B.	15	19	W.	24	15
B.	10	19	W.	17	13	B.	9	14	W.	18	9
B.	5	14	W.	22	17	B.	7	10	W.	27	24
B.	19	23	W.	26	19	B.	16	23	W.	31	26
B.	14	18	W.	26	19	B.	18	22	W.	17	14
B.	10	17	W.	21	14	B.	3	7	W.	14	9
B.	4	8	W.	9	5	B.	8	11	W.	32	27
B.	6	10	W.	27	23	B.	11	15	W.	23	19
B.	7	11	W.	24	20	B.	15	24	W.	28	19
B.	11	15	W.	30	25	B.	15	24	W.	25	18
B.	1	6	W.	5	1	B.	6	13	Drawn.		

Game

Game 4.

W. from	22 to	17	B.	11	15	W.	25	22	B. 9	13
W.	23	18	B.	6	9	W.	18	11	B. 8	15
W.	27	23	B.	9	14	W.	30	25	B. 5	9
W.	24	19	B.	15	24	W.	28	19	B. 7	11
W.	22	18	B.	13	22	W.	26	17	B. 3	8
W.	32	28	B.	11	15	W.	18	11	B. 8	24
W.	28	19	B.	4	8	W.	17	13	B. 2	6
W.	25	22	B.	8	11	W.	31	26	B. 11	16
W.	22	17	B.	14	18	W.	23	7	Drawn.	

Game 5.

W. from	22 to	18	B.	11	16	W.	25	22	B. 10	14
W.	29	25	B.	16	20	W.	24	19	B. 8	11
W.	19	15	B.	4	8	W.	22	17	*B. 7	10
W.	25	22	B.	10	19	W.	17	10	B. 6	15
W.	23	7	B.	2	11	W.	21	17	B. 1	6
W.	17	13	B.	3	7	W.	28	24	B. 12	16
W.	26	23	B.	8	12	W.	23	19	B. 16	23
W.	31	26	B.	7	10	W.	26	19	B. 11	16
W.	18	11	B.	16	23	W.	27	18	B. loſes.	

* Black loſes the Game by this Move.

Game 6.

W. from	22 to	18	B.	11	16	W.	25	22	B. 10	14
W.	29	25	B.	8	11	W.	24	19	B. 16	20
W.	19	15	B.	4	8	W.	22	17	B. 12	16
W.	17	10	B.	7	14	W.	26	22	B. 2	7
W.	28	24	B.	16	19	W.	23	16	B. 14	23
W.	27	18	B.	20	27	W.	31	24	B. 11	27
W.	32	23	B.	7	10	W.	15	11	B. 8	15
W.	18	11	B.	10	15	W.	21	17	B. 3	7
W.	11	2	B.	9	13	W.	2	9	B. 5	21
W.	23	18	B.	15	19	W.	18	14	B. 19	23
W.	22	18	B.	13	17	W.	18	15	B. 23	27
W.	25	22	B.	21	30	W.	14	10	B. 30	26
W.	23	19	B.	26	23	W.	19	16	B. 23	18
W.	16	11	Drawn Game.							

Game 7.

B. from 11 to 15		W.	22	18	B.	15	22	W.	25	18
B.	8 11	W.	29	25	B.	4	8	W.	25	22
B.	12 16	W.	24	20	B.	10	15	*W.	27	24
B.	16 19	W.	23	16	B.	15	19	W.	24	15
B.	9 14	W.	18	9	B.	11	25	W.	32	27
B.	5 14	W.	27	23	B.	6	10	W.	16	12
B.	8 11	W.	28	24	B.	25	29	W.	30	25
B.	29 22	W.	26	17	B.	11	15	W.	20	16
B.	15 18	W.	24	20	B.	18	27	W.	31	24
B.	14 18	W.	16	11	B.	7	16	W.	20	11
B.	18 23	W.	11	8	B.	23	27	W.	8	4
B.	27 31	W.	24	20	B.	27	23	W.	8	11
B.	23 18	W.	11	8	B.	18	25	W.	loses.	

* White loses the Game by this Move.

Game 8.

B. from 11 to 15		W.	22	18	B.	15	22	W.	25	18
B.	8 11	W.	29	25	B.	4	8	W.	25	22
B.	12 16	W.	24	20	B.	10	15	W.	21	17
B.	7 10	W.	27	24	B.	8	12	W.	17	13
B.	9 14	W.	18	9	B.	5	14	W.	24	19
B.	15 24	W.	28	19	B.	14	17	W.	32	27
B.	10 14	W.	27	24	B.	3	7	W.	30	25
B.	6 9	W.	13	6	B.	1	10	W.	22	13
B.	14 18	W.	23	14	B.	16	30	W.	25	21
B.	10 17	W.	21	14	B.	30	25	W.	14	9
B.	11 15	W.	9	6	B.	2	9	W.	13	6
B.	15 18	W.	6	2	B.	7	10	W.	2	6

Drawn Game.

Game 9.

B. from 11 to 15	W. 22 18	B. 15 22	W. 25 18
B. 8 11	W. 29 25	B. 4 8	W. 25 22
B. 10 15	W. 24 20	B. 12 16	W. 21 17
B. 7 10	W. 17 13	B. 8 12	W. 28 24
B. 10 14	W. 23 19	B. 16 23	W. 26 10
B. 14 23	W. 27 18	B. 6 15	W. 13 6
B. 1 10	W. 31 26	B. 5 9	W. 26 23
B. 9 13	W. 23 19	B. 13 17	W. 22 13
B. 15 22	W. 32 28	B. 10 14	W. 19 16
B. 12 19	W. 24 8	B. 3 12	W. 13 9
B. 14 18	W. 28 24	B. 18 23	W. 24 19
B. 23 27	W. 19 15	B. 27 32	W. 15 11
B. 32 27	W. 9 5	B. 27 23	W. 5 1

Drawn Game.

Game 10.

W. from 22 to 18	B. 11 15	W. 18 11	B. 8 15
W. 21 17	B. 4 8	W. 17 13	B. 8 11
W. 25 22	B. 9 14	W. 29 25	B. 5 9
W. 23 19	B. 14 17	W. 27 23	B. 17 21
W. 22 17	B. 11 16	W. 25 22	B. 16 20
W. 19 16	B. 20 27	W. 31 24	B. 12 19
W. 23 16	B. 10 14	W. 17 10	B. 7 14
W. 24 19	B. 15 24	W. 28 19	B. 1 5
W. 22 17	B. 14 18	W. 26 23	B. 18 27
W. 32 23	B. 6 10	W. 13 6	B. 2 9
W. 17 13	B. 9 14	Drawn Game.	

Game 11.

B. from 11 to 15	W. 22 17	B. 9 13	W. 17 14
B. 10 17	W. 21 14	B. 8 11	W. 24 19
B. 15 24	W. 28 19	B. 11 16	W. 25 21
B. 6 9	W. 29 25	B. 9 18	W. 23 14
B. 16 23	W. 26 19	B. 4 8	W. 25 22
B. 8 11	W. 22 18	B. 11 16	W. 27 23
B. 16 20	W. 31 27	B. 13 17	W. 30 26
B. 1 6	*W. 18 15	B. 20 24	W. 27 20
B. 7 10	W. 14 7	B. 2 27	W. 21 14
B. 6 9	W. 32 23	B. 9 27	W. loses.

* White loses the Game by this Move.

Game

Game 12.

B. from	11 to	15	W.	22	17	B.	9	13	W.	17	14
B.	10	17	W.	21	14	B.	8	11	W.	24	19
B.	15	24	W.	28	19	B.	11	16	W.	25	21
B.	6	9	W.	29	25	B.	9	18	W.	23	14
B.	16	23	W.	26	19	B.	4	8	W.	25	22
B.	8	11	W.	22	18	B.	11	16	W.	27	23
B.	16	20	W.	31	27	B.	13	17	W.	30	26
B.	1	6	*W.	19	16	B.	12	19	W.	23	16
B.	6	9	W.	18	15	B.	9	18	W.	21	14
B.	7	11	W.	15	8	B.	3	19	W.	27	23
B.	18	27,	&c.			Drawn Game.					

* This is the Move White ſhould have made in the 11th Game.

Game 13.

B. from	11 to	16	W.	22	18	B.	16	19	W.	23	16
B.	12	19	W.	24	15	B.	10	19	W.	25	22
B.	9	14	W.	18	9	B.	5	14	W.	22	17
B.	7	10	W.	27	24	B.	2	7	W.	24	15
B.	10	19	W.	17	10	B.	7	14	W.	32	27
B.	3	7	W.	27	24	B.	7	10	W.	24	15
B.	10	19	W.	31	27	B.	8	11	W.	29	25
B.	6	10	W.	27	23	B.	11	16	W.	25	22
B.	10	15	W.	22	17	B.	15	18,	&c.		

Drawn game.

Game 14.

W. from	22 to	18	B.	10	15	W.	25	22	B.	6	10
W.	29	25	B.	10	14	W.	24	19	B.	15	24
W.	28	19	B.	11	16	W.	18	15	B.	7	11
W.	22	18	B.	16	20	W.	26	22	B.	11	16
W.	15	10	B.	9	13	W.	18	9	B.	5	14
W.	19	15	B.	16	19	W.	23	16	B.	12	19
W.	22	18	B.	14	23	W.	27	18	B.	2	6
W.	25	22	B.	19	24	W.	18	14	B.	24	27
W.	32	23	B.	8	11	W.	15	8	B.	4	11
W.	23	18	B.	6	15	W.	14	10	B.	20	24
W.	18	14	B.	11	16	W.	30	26	B.	16	20
W.	22	17	B.	13	22	W.	26	17	Drawn.		

Game 15.

W. from 22 to 18	B. 11 16	W. 25 22	B. 8 11
W. 29 25	B. 4 8	W. 18 14	B. 10 17
W. 21 14	B. 9 18	W. 23 14	B. 6 10
W. 22 18	B. 10 17	W. 25 21	B. 1 6
W. 21 14	*B. 6 10	W. 24 20	B. 10 17
W. 18 15	B. 11 18	W. 20 4	B. 17 21
W. 4 8	B. 5 9	W. 28 24	B. 9 13
W. 27 23	B. 18 27	W. 32 23	B. 12 16
W. 8 12	B. 16 20	W. 26 22	B. 20 27
W. 31 24	B. 7 11	W. 23 18	B. 3 7
W. 18 14	B. 11 15	W. 14 9	B. 7 10
W. 12 8	B. 14 18	W. 8 11	B. 14 18
W. 24 20	B. 18 25	W. 11 18	B. 25 29
W. 9 5	B. loſes.		

* Black loſes the game by this Move.

Game 16.

B. from 11 to 16	W. 22 18	B. 8 11	W. 25 22
B. 4 8	W. 29 25	B. 16 19	W. 24 15
B. 10 19	W. 23 16	B. 12 19	W. 21 17
B. 9 14	W. 17 10	B. 7 23	W. 27 18
B. 11 16	W. 18 15	B. 9 6	W. 22 17
B. 1 6	W. 26 22	B. 3 7	W. 22 18
B. 7 10	W. 17 13	B. 16 20	W. 25 22
B. 9 14	W. 18 9	B. 5 14	W. 22 18
B. 14 23	W. 31 27	B. 8 12, &c.	

Drawn Game.

Game 17.

B. from 11 to 15	W. 22 17	B. 15 19	W. 24 15
B. 10 19	W. 23 16	B. 12 19	W. 25 22
B. 7 10	W. 27 24	B. 10 15	W. 22 18
B. 15 22	W. 24 15	*B. 3 7	W. 30 25
B. 9 13	W. 25 18	B. 13 22	W. 26 17
B, 7 10	W. 31 26	B. 10 19	W. 32 27
B. 2 7	W. 17 14	B. 7 11	W. 27 24
B. 11 15	W. 18 11	B. 8 15	W. 14 10
B. 6 9	W. 10 7	B. 9 14	W. wins.

* Black loſes the Game by this Move.

Game 18.

B. from 11 to 15	W. 22 17	B. 15 19	W. 24 15
B. 10 19	W. 23 16	B. 12 19	W. 25 22
B. 7 10	W. 27 24	B. 10 15	W. 22 18
B. 15 22	W. 24 15	*B. 9 13	W. 26 23
B. 8 11	W. 15 8	B. 4 11	W. 28 24
B. 3 7	W. 24 19	B. 6 10	W. 17 14
B. 10 17	W. 21 14	B. 1 6	W. 30 25
B. 6 10	W. 25 18	B. 10 17	W. 19 15
B. 11 16	W. 15 11	B. 7 10	

Drawn Game.

* This is the Move Black ought to have made in the last Game.

Game 19.

W. from 22 to 18	B. 11 16	W. 25 22	B. 10 14
W. 24 19	B. 8 11	W. 27 24	B. 16 20
W. 31 27	B. 11 16	W. 19 15	*B. 16 19
W. 23 16	B. 12 19	W. 15 11	B. 14 23
W. 24 15	B. 7 16	W. 26 12	B. 4 8
W. 28 24	B. 9 14	W. 24 19	B. 2 7
W. 30 26	B. 14 18	W. 26 23	B. 18 25
W. 29 22	B. 7 10	W. 23 18	B. 5 9
W. 27 24	B. 20 27	W. 32 23	B. loses.

* Black loses the Game by this Move.

Game 20.

W. from 22 to 18	B. 11 16	W. 25 22	B. 10 14
W. 24 19	B. 8 11	W. 27 24	B. 16 20
W. 31 27	B. 4 8	W. 29 25	B. 11 16
W. 19 15	B. 7 11	W. 24 19	B. 9 13
W. 18 9	B. 5 14	*W. 22 18	B. 1 5
W. 18 9	B. 5 14	W. 26 22	B. 11 18
W. 22 15	B. 3 7	W. 28 24	B. 7 10
W. 30 26	B. 14 17	W. 21 7	B. 2 18
W. 23 14	B. 16 30	W. 14 9	

Black Wins.

* White loses the Game by this Move.

Game 21.

W. from 22 to 18	B. 10 14	W. 24 19	B. 11 16
W. 27 24	B. 16 20	W. 31 27	B. 8 11
W. 25 22	B. 4 8	W. 29 25	B. 11 16
W. 19 15	B. 7 11	W. 24 19	B. 9 13
W. 18 9	B. 5 14	W. 28 24	B. 11 18
W. 22 15	B. 6 10	W. 15 6	B. 1 10
W. 26 22	B. 3 7	W. 22 18	B. 14 17
W. 21 14	B. 10 17	W. 18 14	B. 8 11
W. 14 9	B. 7 10	W. 9 5	B. 10 14
W. 25 21	B. 17 22	W. 5 1	B. 22 25
W. 1 5	B. 25 29	W. 5 9	B. 13 17
W. 9 18	B. 17 22	W. 18 25	B. 29 22
W. 19 15	B. 11 18	W. 23 14	B. 2 6
W. 21 17,	&c. Drawn.		

Game 22.

B. from 11 to 15	W. 23 19	B. 9 13	W. 26 23
B. 8 11	W. 23 18	B. 4 8	W. 27 23
B. 6 9	W. 30 26	B. 9 14	W. 18 9
B. 5 14	W. 32 27	B. 1 5	W. 19 16
B. 12 19	W. 23 16	B. 11 20	W. 22 17
B. 13 22	W. 25 4	B. 5 9	W. 29 25
B. 9 13	W. 25 22	B. 14 17	W. 21 14
B. 10 17	W. 26 23	B. 17 26	W. 31 22
B. 7 11	W. 24 19	B. 2 7	

White wins.

Game 23.

B. from 11 to 15	W. 23 19	B. 8 11	W. 22 17
B. 9 14	W. 25 22	B. 11 16	W. 24 20
B. 16 23	W. 27 11	B. 7 16	W. 20 11
B. 3 7	W. 28 24	B. 7 16	W. 24 19
B. 16 23	W. 26 19	B. 4 8	W. 30 26
B. 8 11	W. 26 23	B. 11 15	W. 32 28
B. 15 24	W. 28 19	B. 5 9	W. 29 25
B. 9 13	W. 31 27	B. 1 5	W. 27 24
B. 6 9	W. 24 20	B. 2 7	W. 20 16
B. 14 18	W. 23 14	B. 9 18	W. 22 6
B. 13 29	W. 6 2	B. 7 10	W. 16 11
B. 10 14	W. 2 6	B. 29 25,	&c.

Drawn Game.

Game 24.

B. from 11 to 15	W. 22 17	B. 8 11	W. 25 22
B. 9 13	*W. 29 25	B. 15 18	W. 23 14
B. 11 15	W. 24 19	B. 15 24	W. 28 19
B. 4 8	W. 26 23	B. 8 11	W. 23 18
B. 6 9	W. 27 24	B. 1 6	W. 32 28
B. 11 15	W. 18 11	B. 9 18	W. 22 15
B. 13 29	W. 11 8	B. 29 25	W. 31 26
B. 5 9, &c.	White loses.		

* White loses the Game by this Move.

Game 25.

B. from 11 to 16	W. 22 18	B. 10 14	W. 25 22
B. 8 11	W. 29 25	B. 4 8	W. 24 20
B. 16 19	W. 23 16	B. 14 23	W. 27 18
B. 12 19	W. 32 27	B. 9 14	W. 18 9
B. 5 14	*W. 22 17	B. 19 23	W. 26 19
B. 8 12	W. 17 10	B. 6 24	W. 28 19
B. 11 16	W. 20 11	B. 7 32,	&c.

White loses.

* White loses the Game by this Move.

Game 26.

B. from 11 to 16	W. 22 18	B. 10 14	W. 25 22
B. 8 11	W. 29 25	B. 4 8	W. 18 15
B. 11 18	W. 22 15	B. 16 20	W. 26 22
B. 14 18	W. 23 14	B. 9 18	W. 24 19
B. 7 11	W. 27 24	B. 20 27	W. 32 14
B. 11 18	W. 22 15	B. 6 10	W. 14 7
B. 2 18	W. 28 24	B. 3 7	W. 21 17
B. 7 10	W. 17 14	B. 10 17	W. 25 22
B. 18 25	W. 30 14	B. 8 11, &c.	

Drawn Game.

Game 27.

W.	from 22	to 17	B.	11	15	W.	23	19	B.	8	11
W.	25	22	B.	4	8	W.	29	25	B.	9	13
W.	17	14	B.	10	17	W.	19	10	B.	7	14
W.	22	18	B.	14	23	W.	21	14	B.	11	16
W.	27	18	B.	3	7	W.	24	20	B.	16	19
W.	32	27	B.	6	10	W.	25	21	B.	10	17
W.	21	14	B.	1	6	W.	27	24	B.	19	23
W.	26	19	B.	6	10	W.	30	26	B.	10	17
W.	26	22	B.	17	26	W.	31	22	B.	2	6
W.	18	15	B.	7	10	W.	20	16	B.	10	14
W.	15	11	B.	8	15	W.	19	1	B.	12	19
W.	24	15	B.	14	18	W.	1	6	Drawn.		

Game 28.

W.	from 22	to 17	B.	11	15	W.	23	19	B.	8	11
W.	25	22	B.	9	13	W.	17	14	B.	10	17
W.	19	10	B.	7	14	W.	29	25	B.	2	7
W.	27	23	B.	6	10	W.	31	27	B.	4	8
W.	24	20	B.	12	16	W.	27	24	B.	8	12
W.	24	19	B.	5	9	W.	19	15	B.	10	19
W.	23	18	B.	14	23	W.	21	5	B.	7	10
W.	25	21	B.	10	15	W.	28	24	B.	19	28
W.	26	10	B.	16	19	W.	21	17,	&c.		

Drawn Game.

Game 29.

W.	from 22	to 17	B.	11	15	W.	23	19	B.	8	11
W.	25	22	B.	9	13	W.	17	14	B.	10	17
W.	19	10	B.	7	14	W.	29	25	B.	2	7
W.	27	23	B.	11	16	W.	31	27	B.	16	20
W.	23	18	B.	14	23	W.	21	14	B.	6	9
W.	27	18	B.	20	27	W.	32	23	B.	4	8
W.	23	19	B.	8	11	W.	28	24	B.	11	16
W.	24	20	B.	16	23	W.	26	19	B.	1	6
W.	25	21	B.	6	10	W.	21	17	B.	7	11
W.	14	7	B.	3	10	W.	19	16	B.	12	19
W.	17	14	B.	10	26,	&c.			Drawn.		

Game 30.

B.	from 11 to	15	W.	22	17	B.	8	11	W.	23	19
B.	4	8	W.	25	22	B.	9	13	W.	17	14
B.	10	17	W.	19	10	B.	7	14	W.	29	25
B.	2	7	W.	27	23	B.	11	16	W.	22	18
B.	6	10	W.	18	9	B.	5	14	W.	24	20
B.	16	19	W.	23	16	B.	12	19	W.	32	27
B.	1	6	W.	27	23	B.	8	12	W.	23	16
B.	12	19	W.	31	27	B.	14	18	W.	21	14
B.	10	17	W.	25	22	B.	18	25	Drawn.		

The End of the Games.

Critical Situations to win Games.

First Situation.

No. 21 a black man, 25 a black king, 26 27 white kings, and either to move.

B. from	25 to	22	W.	27	23	B.	29	25	W.	23	18
B.	25	29	W.	18	22	B.	21	25	W.	26	30

Second Situation.

No. 1, 2, black kings, 10 11 white kings, 5 a white man, and either to play.

W. from	10 to	14	B.	2	6	W.	14	17	B.	6	9
W.	17	13	B.	9	6	W.	11	16	B.	6	2
W.	16	19	B.	2	6	W.	19	23	B.	6	2
W.	13	9	B.	1	6	W.	23	18	B.	6	13
W.	18	14	B.	13	9	W.	14	10, &c.			

Set the Men as before.

B. from	2 to	6	W.	11	15	B.	6	9	W.	15	18
B.	9	6	W.	10	14	B.	6	9	W.	14	17
B.	9	13	W.	18	22	B.	13	9	W.	17	13
B.	9	6	W.	22	18	B.	6	2	W.	13	9
B.	1	6	W.	18	14	B.	6	13	W.	5	1

Third Situation.

No. 1, 2, black kings, 3 a black man, 9, 10, 11, white kings, 12 a white man, and black to play.

B. from	1 to	5	W.	9	13	B.	5	1	W.	11	15
B.	2	6	W.	10	14	B.	6	2	W.	14	9
B.	1	6	W.	9	5	B.	6	1	W.	15	11
B.	2	6	W.	11	7	B.	3	10	W.	5	9

Fourth

Fourth Situation.

No. 5 a white king, 21 a white man, 6 10 black kings, black being to move may win.

B. from	6 to	1	W.	5	9	B.	10	15	W.	9	5
B.	15	18	W.	5	9	B.	1	5	W.	9	6
B.	18	15	W.	21	17	B.	5	1	W.	6	9
B.	15	18	W.	9	5	B.	18	22	W.	17	14
B.	1	6	W.	5	1	B.	6	2	W.	14	10
B.	22	18	W.	1	5	B.	18	14			

Place the Men as before.

B. from	6 to	1	W.	5	9	B.	10	15	W.	21	17
B.	15	18	W.	17	13	B.	18	15	W.	9	14
B.	1	5	W.	14	17	B.	15	10	W.	17	22
B.	10	14	W.	22	25	B.	5	1	W.	25	22
B.	1	6	W.	22	25	B.	6	10	W.	25	30
B.	10	15	W.	30	25	B.	15	18,	&c.		

Fifth Situation.

No. 1 a white king, 30 a white man, 9 10 black kings, and black being to play may win.

B. from	9 to	6	W.	1	5	B.	6	1	W.	5	9
B.	1	5	W.	9	13	B.	10	14	W.	13	9
B.	14	18	W.	9	6	B.	18	15	W.	30	25
B.	15	18	W.	25	21	B.	5	1	W.	6	9
B.	18	22	W.	9	5	B.	1	6	W.	5	1
B.	6	9	W.	1	5	B.	9	14	W.	5	1
B.	22	18	W.	1	5	B.	18	15	W.	5	1
B.	15	10	W.	1	5	B.	10	6	W.	5	1
B.	14	10	W.	1	5						

Now black has the fourth situation, and must consequently win.

Sixth Situation.

No. 22, 27, white kings, 18 a white man, 5 a black king, 20, 21, black men, and white being to play may win.

W. from	18 to	14	B.	5	1	W.	14	9	B.	1	5
W.	22	17	B.	5	14	W.	17	10	B.	21	25
W.	10	15	B.	25	30	W.	15	19	B.	30	25
W.	27	32	B.	25	22	W.	19	24	B.	20	27
W.	32	23									

Seventh Situation.

No. 6, 24, black kings, 14, 18, 23, white kings, and either to move, white may win.

W. from	18 to	15	B.	6	1	W.	14	9	B.	24	28
W.	23	19	B.	1	5	W.	9	6	B.	28	32
W.	19	24	B.	5	1	W.	24	19, &c.			

Eighth Situation.

No. 1, 22, 16, black men, 13 a black king, 5, 6, 10 white men, 11 white king, and black to play.

B. from	13 to	9	W.	11	20	B.	9	2	W.	20	24
B.	12	16	W.	24	27	B.	16	19	W.	27	32
B.	19	24	W.	32	28	B.	2	6	W.	28	19
B.	6	24									

Situations for Strokes.

First Stroke.

On No. 17 a black man, on No. 30 a black king, 18 27 white kings, and white to play.

W. 18 22 B. 17 26 W. 27 31.

Second Stroke.

No 17 27 white kings, 18 a black man, 29 30 black kings, and white to play.

W. 17 22 B. 18 25 W. 27 23.

Third Stroke.

No. 18 19 white kings, 28 a white man, 31 32 black kings, 20 a black man, and white to move.

W. 19 24 B. 20 27 W. 18 22

Fourth Stroke.

No. 9, 11, 21 black men, 29 a black king, 18, 24, 26, 30 white men, and white to move.

W. 18 14 B. 9 18 W. 26 22 B. 18 25
W. 24 19

Fifth Stroke.

No. 12 21 black men, 27 31 black kings, 20 30 white men, 15 18 white kings, and white to move.

W. 30 26 B. 31 22 W. 18 25 B. 21 30
W. 20 16 B. 12 19 W. 15 31

Sixth Stroke.

No. 7 23 black kings, 9 13 black men, 8, 21, 22 white men, 17 a white king, and white to move.

W. 22 18 B. 13 22 W. 8 3 B. 23 14
W. 3 26

Seventh Stroke.

No. 3, 13, 14, black men, 24 a black king, 15 22 white kings, 19 21 white men, and white to move.

W. 21 17 B. 14 21 W. 15 18 B. 24 15
W. 18 11

Eighth Stroke.

No. 1, 6, 9, black men, 18 a black king, 7 a white king, 13 15 white men, and white to play.

W. 15 10 B. 6 15 W. 13 6 B. 1 10
W. 7 23

Ninth Stroke.

No. 6, 7, white kings, 9 a white man, 5 a black man, 14, 15, black kings, and white to play.

W. 7 10 B. 14 7 W. 6 2 B. 5 14
W. 2 9

Tenth Stroke.

No. 2, 6, 8, 22, black men, 15, 27, 30, 32, white men, and white to play.

W. 15 11 B. 8 15 W. 30 26 B. 22 31
W. 32 28 B. 31 24 W. 28 1

Eleventh Stroke.

No. 6 26 white men, 22 a white king, 7 15 black kings, 21 a black man, and white to play.

W. 22 25 B. 21 30 W. 6 2 B. 30 23
W. 2 27

Twelfth Stroke.

No. 2 a black man, 27 31 black kings, 10 a white man, 14 19 white kings, and white to move.

W. 10 7 B. 2 11 W. 19 15 B. 11 18
W. 14 32

Thirteenth Stroke.

No. 3 13 black men, 25 26 black kings, 11 a white man, 15 16 white kings, and white to move.

W. 11 7 B. 3 19 W. 16 21

Fourteenth Stroke.

No. 3 a black man, 26 27 black kings, 11 a white man, 15 16 white kings, and white to move.

W. 11 8 W. 3 19 W. 15 22

Fifteenth Stroke.

No. 1, 3, 5, black men, 25 a black king, 10, 14, 17, white men, 13 a white king, and white to move.

W. 10 6 B. 1 10 W. 14 7 B. 3 10
W. 17 14 B. 10 17 W. 13 29, &c.

Sixteenth Stroke.

No. 1, 6, 7, 10, 12, 14, 15, black men, 19, 20, 21, 22, 23, 26, 30, white men, and white to move.

W. 20 16 B. 15 24 W. 22 18 B. 12 19
W. 18 2

Seventeenth

Seventeenth Stroke.

No. 2, 3, 16, 23, black men, 14 a black king, 1 5 white kings, 9 31 white men, and black to move.

B. 23 27 W. 31 24 B. 16 19 W. 24 15
B. 14 10 W. 15 6 B. 3 7 W. 29 25
B. 7 10 W. 25 22 B. 10 14

Eighteenth Stroke.

No. 10, 13, 17, black men, 27 a black king, 19, 22, 26, 30, white men, and white to play.

W. 26 23 B. 17 26 W. 19 16 B. 27 18
W. 30 7

Nineteenth Stroke.

No. 1, 6, 10, 19, 20, black men, 13, 15, 27, 28, 31, white men, and white to play.

W. 13 9 B. 6 13 W. 15 6 B. 1 10
W. 27 24 B. 20 27 W. 31 6

Twentieth Stroke.

No. 1, 3, 5, 6, 7, 12, 20, 21, black men, 14, 15, 19, 23, 26, 27, 30, 32, white men, and white to play.

W. 30 25 B. 21 30 W. 14 10 B. 7 14
W. 19 16 B. 12 19 W. 23 16 B. 30 23
W. 27 2

Twenty-first Stroke.

No. 3, 6, 10, 13, 14, 17, 19, black men, 7, 20, 21, 22, 26, 30, white men, and black to move.

B. 19 23 W. 26 19 B. 17 26 W. 30 23
B. 14 18 W. 23 14 B. 10 17 W. 21 14
B. 3 17

Twenty-second Stroke.

No. 2, 7, 10, 11, 12, 13, 14, 21, black men, 19, 20, 22, 23, 26, 30, 31, 22, white men, and white to move.

W. 20 16 B. 11 20 W. 19 15 B. 10 19
W. 23 16 B. 12 19 W. 22 17 B. 13 22
W. 26 3

Twenty-third Stroke.

No. 3, 5, 8, 10, 11, 15, 16, 22, black men, 17, 18, 20, 27, 28, 29, 31, 32, white men, and white to move.

W. 31 26 B. 22 31 W. 18 14 B. 31 24
W. 14 7 B. 3 10 W. 28 3

Twenty-fourth Stroke.

No. 5 12 black men, 14, 29, 32, black kings, 8, 9, 30, 31, white men, 15 a white king, and white to move.

W. 31 27 B. 32 23 W. 30 25 B. 29 22
W. 15 10 B. 14 7 W. 8 3 B. 5 14
W. 3 19

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